Mastering the Basic Math Facts in Addition and Subtraction

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Strategies, Activities & Interventions to Move Students Beyond Memorization

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HEINEMANN
Portsmouth, NH

Heinemann
361 Hanover Street
Portsmouth, NH 03801–3912
www.heinemann.com

Offices and agents throughout the world

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Library of Congress Cataloging-in-Publication Data
O'Connell, Susan.
Mastering the basic math facts in addition and subtraction : strategies, activities & interventions to move students beyond memorization / Susan O'Connell and John SanGiovanni.
p. cm.
Includes bibliographical references.
ISBN-13: 978-0-325-02963-4
ISBN-10: 0-325-02963-6
1. Addition—Study and teaching (Primary). 2. Subtraction—Study and teaching (Primary). I. SanGiovanni, John. II. Title.
QA115 .O33 2011
372.7'2—dc22 2010046911

Editor: Victoria Merecki
Production: Victoria Merecki
Cover and interior designs: Palmer Creative Group
Composition: Publishers' Design and Production Services, Inc.
Manufacturing: Steve Bernier

Printed in the United States of America on acid-free paper

15 14 13 12 11 VP 1 2 3 4 5

Dedication

To sweet Bailey with love

S.O.

To Kay, thank you for being a mentor and friend

J.S.

Contents

Foreword xiii

Acknowledgments xv

Introduction 1

What are basic facts? 1

What constitutes *mastery* of basic facts? 2

Why do we need to know basic facts? 2

Why is it important to understand math facts? 3

How can we help students master basic math facts? 5

How can this book help you? 7

How is this book organized? 8

What is the teaching sequence of math facts within this book? 9

Why are activities and resources on a CD? 12

Resource for Professional Learning Communities 13

Our Goal 13

CHAPTER 1 Understanding Addition and Subtraction 15

Exploring Big Ideas to Develop Math Fact Strategies 16

Building Understanding Prior to Focusing on Fluency 18

Using Models to Represent Addition and Subtraction 18

Exploring the Concepts of Addition and Subtraction Through Problems 23

Exploring Operations Through Children's Literature 25

Partner Explorations 26

The Classroom Environment 27

Beginning with Understanding 29

CHAPTER 2 Plus One and Plus Two 31

Focusing on the Big Ideas 32

Understanding +1, +2 Facts 34

Literature Link: *Mouse Count* 34

Exploring the Facts: Number Strips to Visualize 1 More and 2 More 38

Supporting All Learners 39

Building Automaticity 41

Targeted Practice 42

Monitoring Progress 45

Connecting to Subtraction 46

CHAPTER 3 Adding Zero 51

Focusing on the Big Ideas 52

Understanding +0 Facts 53

Literature Link: *Gray Rabbit's 1, 2, 3* 53

Exploring the Facts: Acting Out +0 Facts 54

Supporting All Learners 56

Building Automaticity 58

Targeted Practice 58

Monitoring Progress 60

Connecting to Subtraction 60

CHAPTER 4 Adding Ten 63

Focusing on the Big Ideas 64

Understanding +10 Facts 65

Literature Link: *If You Give a Mouse a Cookie* 65

Exploring the Facts: Using Double Ten-Frames 66

Supporting All Learners 68

Building Automaticity 70

Targeted Practice 70

Monitoring Progress 72

Connecting to Subtraction 72

CHAPTER 5 Doubles 77

Focusing on the Big Ideas 78

Understanding Doubles 78

Literature Link: *Double the Ducks* 78

Exploring the Facts: Creating Equal Sets 81

Supporting All Learners 82

Building Automaticity 85

Targeted Practice 85

Monitoring Progress 88

Connecting to Subtraction 88

CHAPTER 6 Making Ten 93

Focusing on the Big Ideas 94

Understanding Making Ten 95

Literature Link: *Ten Apples up on Top!* 95

Exploring the Facts: The Cupcake Problem 97

Supporting All Learners 98

Building Automaticity 101
Targeted Practice 101
Monitoring Progress 106
Connecting to Subtraction 107

CHAPTER 7 Using Tens 111
Focusing on the Big Ideas 113
Understanding Using Tens 114
Literature Link: *Diary of a Worm* 114
Exploring the Facts: Counters In and Out 117
Supporting All Learners 118
Building Automaticity 121
Targeted Practice 121
Monitoring Progress 123
Connecting to Subtraction 124

CHAPTER 8 Using Doubles 127
Focusing on the Big Ideas 128
Understanding Using Doubles 129
Literature Link: *Fish Eyes* 129
Exploring the Facts: Doubles and One More 131
Supporting All Learners 132
Building Automaticity 134
Targeted Practice 134
Monitoring Progress 136
Connecting to Subtraction 137

One Lonely Fact 140

Retaining Math Facts 140

Automatic Recall of Facts 142

Conclusion 143

References 145

Professional References 145

Children's Literature 145

Professional Learning Communities Study Guide 147

Guiding Questions 149

A Guide to the CD-ROM 155

Organization of the CD Files 155

Teaching Resources 155

Teaching Tools 155

Fact Cards 155

Assessment Tools 156

Modifications to CD Activities 156

Foreword

Being able to add and subtract within 20 and multiply and divide within 100 is essential during the early years of schooling, and the basic facts of addition/subtraction and multiplication/division are a critical baseline, not only then but also during later work with fractions, decimals, ratio, proportion, and more. Foundational? You better believe it. Essential? Absolutely.

That said, the basic facts are also problematic. The goal is for most students to know, fluently, and with automaticity, the addition/subtraction facts, typically by the end of second grade and the multiplication/division facts, typically by the end of third grade. But far too many teachers are unable to help their students reach these goals. "Not this year," they may mutter, or, "Not all my students," or worse, "Not ever." Why is fluency with the basic facts such a challenge for so many students? In our digit-conscious culture students can spout off multiple phone and pin numbers, but not the product of 6×7! I meet and work with middle school students who are still wondering about 8×7 or $48 \div 6$ and other basic facts. Why do far too many students fail to realize that the commutative property means that $9 + 7$ and $7 + 9$ get you to the same place, 16? This drives us all crazy! Have we neglected the basics? Is this about just having students memorize the facts? No, and no!

Over twenty years ago the *Curriculum and Evaluation Standards for School Mathematics* noted that "children should master the basic facts of arithmetic that are essential components of fluency with paper-pencil and mental computation and with estimation" (47).[1] The National Research Council's *Adding It Up* dedicates almost ten pages to synthesizing the research dealing with basic fact acquisition.[2] More recently, the *Final Report of the National Mathematics Advisory Panel* points out that computational proficiency with whole number operations depends on the practice (I prefer the term *rehearsal*) necessary to develop automatic recall of addition/subtraction and multiplication/division facts.[3] Nurturing computational facility in elementary school requires that students be fluent with the basic facts of arithmetic. How do we get this done?

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[1]National Council of Teachers of Mathematics. 1989. *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

[2]National Research Council. 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

[3]National Mathematics Advisory Panel. 2008. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Over the years teachers have tried and continue to use a myriad of practice activities—oral and written exercises, games, and classroom and homework assignments, many of them now via the Internet. At last we have a more effective option—Susan O'Connell and John SanGiovanni's *Mastering the Basic Facts in Addition and Subtraction: Strategies, Activities & Interventions to Move Students Beyond Memorization* and *Mastering the Basic Facts in Multiplication and Division: Strategies, Activities & Interventions to Move Students Beyond Memorization*. What a find!

Based on Thornton's pioneering work[4] emphasizing how thinking strategies facilitate fact acquisition, both books present activities that develop facility with the basic facts by building a conceptual understanding of the operations; following a teaching sequence designed to develop a sense of number using fact strategies and the commutative property; and using representational models and context-based problem solving. (The activities that link facts to their conceptual representations are also powerful diagnostic tools.) But there's more—related children's literature, partner activities, a professional-learning-community study guide. All these components add up to resources that engage students, from beginning activities that promote an understanding of arithmetic concepts, through fluency with the basic facts.

One final consideration: these books will be very helpful to teachers whose students' mathematical knowledge require some level of intervention. The powerful instructional opportunities these books provide not only make sense but also meet one of the key recommendations of the What Works Clearinghouse's Practice Guide *Assisting Students Struggling with Mathematics*.[5]

These books won't end up on a shelf at the back of your room. (And if you are a third/fourth-grade teacher you will probably need both of them.) You'll use them every day. You'll carry them home with you and talk about them in the faculty lounge. Just as the basic facts are "must haves" on the path to computational fluency, these books are "must haves" to help you navigate the route.

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4 Thornton, C.A. 1978. "Emphasizing Thinking Strategies in Basic Fact Instruction." *Journal for Research in Mathematics Education*. 16: 337–355.

5 Gersten et al. 2009. *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*. Washington, DC: Institute of Education Sciences.

Acknowledgments

Many thanks to the students whose conversations about math facts inspired and excited us as we worked on this manuscript. Thanks to the following students who contributed work samples or allowed their photographs to appear within this book: Samantha Anderson, Jack Bacon, Julia Basket, Riley Benson, Justus Black, Krysta Carte, Eric Chen, Mason Chizhik, Esme Chu, Anessa Coleman, Ben Dyer, Nolan Dyer, Eric Ellis, Satya Emerick, Jonah Eng, Maximillian Fagerstrom, Anna Farley, Maya Frye, Emily Gomez, Maggie Inskeep, Ashley Jeon, Megan Keeley, Christopher Kennedy, Alexandra Liberto, DJ Lindahl, Emerson Mako, Caitlyn McDonald, Brooke Miller, Joshua Morton, Adam Mostafa, Brooke Naidu, Dylan Nguyen, Josh Oberly, Rona Okojie, Zion Olibris, Allison Peay, Jamie Roberts, Hannah Williams, Nushrat–I Rabban, Jack Ryder, Annie Schinkai, Sydney Schinkai, Oscar Schoenfelder, Deryn Schoenfelder, Lauren Stipe, Delaney Thompson, Laya Vanga, and Jilienne Widener. It was a pleasure watching them investigate math facts.

We appreciate the collaboration of colleagues in gathering materials for this book, in particular the following teachers, math coaches, and supervisors who welcomed us into their classrooms, provided insights from their own teaching, or allowed us to listen to the ideas of their students: Elizabeth Bare, Linda Chrest, Connie Conroy, Heather Dyer, Michelle Glenn, Carol Hahn, Renee Holdefelder, Ami Holden, Nena Hupp, Rhonda Inskeep, Arthurlea Kimbrough, Amanda Lewis, Kristen Mangus, Jamie Pickett, Kay Sammons, and Lee Ann Tanis. Thanks, too, to Josie Robles who had the inspiration to use ten-bead counters with her second grade students.

We are grateful to Victoria Merecki, our Heinemann editor, for her guidance on this project from start to finish. We value her insights for the writing and production of the book. In addition, we thank Emily Birch for her vision for this book. Her discussions in the early stages of the book were invaluable.

Special thanks to our families for their patience and understanding during the writing of this book. To Sue's husband Pat, and her children Brendan and Katie, and to John's wife Kristen, and Oscar and Deryn, our warmest thanks for your continued support.

Introduction

As math teachers, we want all of our students to develop a quick recall of single-digit addition and subtraction facts. We label them *basic* math facts because they provide a foundation for math success. We expect that all students will master these basic skills, but that is not a simple goal to achieve. We watch some students effortlessly remember the facts and others struggle with the very same task. And we labor to find just the right strategies and activities to help all students succeed.

As teachers, we are constantly reminded that our students learn in a variety of ways. Although some students have very strong memory skills, others struggle to remember simple facts. Although some students make sense of math concepts on their own, others struggle to connect meaning to simple expressions like 4 + 3. Although some students intuitively use their knowledge of one math fact to find the solution to a related fact, others simply get frustrated and discouraged when they cannot remember a specific sum. Our students are so different, and yet our goal for each of them is the same: to master basic math facts so they have a strong foundation for more complex math skills and procedures. The goal of this book is to explore numerous strategies and activities that support all students in understanding basic addition and subtraction math facts and committing those facts to memory. Whether you are introducing students to basic math facts, reviewing previously taught facts, or providing interventions for students who continue to struggle, this book supplies you with instructional considerations, teaching tips, practical strategies, and numerous classroom-tested activities.

What are basic facts?

In this book, facts with addends of 0–10 are considered basic facts. In some programs, facts with single-digit addends (0–9) are considered basic facts, but because of our emphasis on students' use of strategies when learning basic facts, we have included 10 as an addend. When adding 9 + 6, a student might reason that 10 + 6 = 16, so 9 + 6 is 1 less, or 15. That reasoning is not possible without an understanding of 10 as an addend. To ensure that students have the basic number understandings to reason in this way, addends of 0–10 are addressed. The strategies and activities within this book will focus on mastery of these basic math facts.

What constitutes *mastery* of basic math facts?

In the past, much of mathematics was taught in a drill and practice style. Students were simply asked to memorize their math facts, often without much attention to conceptual understanding. Through worksheets filled with single-digit computations or lengthy flash card sessions, students were asked to memorize addition and subtraction facts. Our goal in today's math classrooms has shifted from memorizing facts and procedures to increased understanding of math skills and concepts. We want our students to be able to do mathematics, but we also want them to understand the math they are doing. We recognize that as math tasks increase in complexity, an understanding of facts, formulas, and algorithms will help them experience continued success. We have not changed our view of the importance of basic math facts. We know that they are a foundational skill without which our students will view even simple math tasks as daunting. We have simply expanded our expectations to include understanding as an important component of our teaching of basic math facts. So, what do we expect of our students? Our goal is both automaticity and understanding. Automaticity is students' ability to effortlessly recall a fact. If students are automatic, they have successfully committed the facts to memory. In addition, we want our students to understand, not simply remember, these important math facts.

Why do we need to know basic math facts?

Ask math teachers what they would like their students to know and be able to do and the recall of basic math facts will undoubtedly rank high on most of their wish lists. Teachers recognize that once their students know 2 + 5, those students are better able to explore 20 + 50 or 22 + 55. Teachers recognize that students will have an easier time finding the solution to \$1.20 + \$1.50 or .21 + .53. These teachers know that their students will be more successful when they are challenged with $\frac{2}{8} + \frac{5}{8}$. As math tasks become more complex, we want our students to have a solid foundation for success.

We have gained insights from brain research about demands on the working brain. As students begin to learn math facts, their brains are focused on these basic computations, but as students become automatic with basic facts,

their brains are able to focus on other aspects of the task like the challenges of place value, decimals, or fractions. Being automatic with basic facts frees the brain to focus on other math processes.

Committing basic math facts to memory speeds up math tasks. As math tasks increase in complexity, they often require multiple steps to find the solution. Addition with three-digit addends and subtraction with decimals are examples of more complex computational tasks. These tasks are time-consuming, and often stressful, for students who must stop to figure out each basic fact along the way. And stopping to determine each fact disrupts the flow of the math procedure. The National Mathematics Advisory Panel (2008) urges that students develop automatic recall of addition and related subtraction facts to be prepared for the study of algebra, in which solving multistep equations is a fundamental task. The panel suggests that by the end of grade 3, students should be proficient with the addition and subtraction of whole numbers (National Mathematics Advisory Panel 2008).

Students who have committed basic math facts to memory are able to perform critical mental math tasks. They estimate answers prior to solving problems so they are able to compare their estimates to the actual answers and determine the reasonableness of their solutions. When playing a playground kickball game, students with mental math skills can determine the new game score after three more runs are scored, or can compare the team scores to figure out how many runs they will need to score in the final inning to win the game. As students spy a dozen cookies, they are able to quickly determine how many will be left after six are eaten, and students with a knowledge of math facts can efficiently find their total score in a family board game. Mastery of basic facts provides the foundation for everyday mental math tasks.

Automaticity is the quick and effortless recall of math facts. No need to count every object, no need to think about related facts, no need to extend patterns. The answer is automatically known. Automaticity with basic facts is a goal for our students, but alone it is not enough. Students must also understand the facts they are being asked to commit to memory.

Why is it important to understand math facts?

The ability to recall items is enhanced when understanding is connected to the task. Memorizing a chain of nonsensical words (e.g., *sat chair red girl a in little the*) is more difficult than memorizing a sentence in which the words

have meaning (e.g., *A little girl sat in the red chair*). Asking students to memorize dozens of number facts can be discouraging and confusing when students view them simply as pairs of numbers. The understanding that 7 + 4 represents the combined total of those two quantities, and that the sum is clearly close to 10, aids our ability to recall the sum.

Students who simply memorize math facts miss a prime opportunity to expand their understanding of equations. Problem solving is the central focus in today's math classrooms. To be a successful problem solver, students must be able to accurately compute answers, but more than that, they must be able to figure out how to build equations that correspond to problem situations.

Kellen's mother asked him to pick up the toy cars he left in the middle of the floor. He put 7 of them on top of his dresser. He put 6 more in his toy box. How many cars did he have?

This problem certainly requires the student to know that 7 + 6 = 13, but even before the student can use his knowledge of math facts to find the answer, he must understand how to build an equation that works with this problem.

> Some cars were placed on the dresser and others were placed in the toy box. I need to know how many he had altogether so I need to add. 7 + 6 will be how I find the answer.

As we discuss the connection between the meaning of the equation and the basic math fact, we are supporting both students' computation skills as well as building a strong foundation for problem solving.

Both the *Common Core State Standards* (National Governors Association Center for Best Practices and Council of Chief State School Officers 2010) and the National Council of Teachers of Mathematics' *Principles and Standards* (2000) emphasize the importance of our students understanding the concepts of addition and subtraction. The *Common Core State Standards* recommend that kindergarten students be given opportunities to explore addition as combining or adding to and subtraction as separating or taking away. Understanding is developed first, with practice for fluency coming later.

How can we help students master basic math facts?

We expect that our students will recall math facts without the need for manipulatives or counting strategies and that the recall will take place within several seconds. Certainly memorization of math facts is our goal, but memorization that comes from ongoing practice and engagement with math facts tasks, not memorization that comes from traditional drill and practice. And we recognize that our students benefit from varied opportunities to explore basic math facts before engaging in practice tasks designed to promote automatic recall. Through hands-on activities and thoughtful discussions, students develop deeper understandings about math facts and cultivate useful strategies related to these basic facts. An instructional approach in which students investigate the conceptual understanding of basic facts, explore strategies to support their understanding of numbers, and then engage in strategic practice in order to automatically recall the facts provides students with a strong and balanced foundation for mastery.

Conceptual Understanding

Understanding operations is fundamental to understanding math facts. Situations are symbolically represented by expressions. $2 + 5$ is more than numbers and symbols. The addition sign helps us understand the relationship between the numbers. $2 + 5$ represents the combined total of two quantities. The understanding of this relationship is critical to making sense of the expression. Through problem posing, hands-on explorations, real-world examples, classroom discussions, and exploring situations from children's literature, students develop deeper understandings of operations. An addition scenario that shows combined groups, or a subtraction story that shows comparisons, helps students strengthen their understanding of operations, and students who understand operations will find that math facts make sense.

Strategic Thinking

There are many ways that students might arrive at an answer to a math fact. When adding $8 + 3$, Jason might simply count every object being added, and Katie might simply remember that $8 + 3 = 11$. Math fact strategies lie somewhere between counting each object and simply memorizing the answer. They are predictable and efficient ways to find answers. Colin might *count on* beginning at 8 to say "8, *9, 10, 11*," while Liam knows that $8 + 2 = 10$ so $8 + 3$

must be 1 more than 10, or 11. Strategies help students find an answer even if they forget what was memorized. Teaching math fact strategies focuses attention on number sense, operations, patterns, properties, and other critical number concepts. These big ideas related to numbers provide a strong foundation for the strategic reasoning that supports mastering basic math facts. For addition and subtraction, understanding the concept of tens, knowing that the order of addends will not affect the sum, and recognizing that various numbers can create the same sum (e.g., $5 + 4 = 9$ and $6 + 3 = 9$) and that there is a unique relationship between those two equations (e.g., in the second expression, the first addend is one more and the second addend is one less) allows students to use their knowledge to build strategies to find sums and differences. Providing opportunities for students to explore math facts through active engagement and meaningful discussions builds their understanding of critical ideas about numbers (Fosnot and Dolk 2001; Gravemeijer and van Galen 2003; Van de Walle 2004) and is an important component of math fact mastery.

Practice for Fluency

Once an understanding of operations has been developed and students have explored strategic reasoning to find solutions to basic math facts, it is time to engage students in meaningful practice so they can commit the facts to memory. Rather than long practice sessions (Remember the lengthy flash card drills of days past?), consider activities that are short in duration but easy to implement, so you can frequently engage students in valuable practice. Scattered practice—five to ten minutes a day, spread throughout the school year—yields powerful results (Marzano, Pickering, and Pollock 2001). And varying the practice activities ensures that students stay motivated and engaged in learning their math facts. Automaticity is achieved through brief, frequent, interactive activities that provide students with repeated exposure to math facts.

Because of the anxiety associated with memorization tasks for many students, the practice tasks in this book do not focus on speed or elimination. Although speed drills or elimination games may be enjoyed by some students, these types of activities often intensify the frustration and anxiety of others. Students who struggle with rote memory tasks, those students who are the reason we include math fact activities in our daily schedule, are just the ones who become discouraged by the speed drills, experience humiliation when they are the first to be eliminated, and sit on the sidelines where they do not get the practice that they need.

There are multiple practice activities within this book that engage students in math fact practice without increasing their anxiety or allowing them to get discouraged. The practice tasks are interactive and hands-on, and provide students with repeated exposure to each set of math facts in a gradual progression in which each new set of facts builds on previous ones. Select the activities that work best for your students. Although some students might find competitive activities fun and motivating, others thrive on collegial tasks. Throughout this book, you will find activity choices to allow you to personalize math facts practice for your students.

How can this book help you?

This book is a practical guide for helping students master addition and subtraction facts. It includes insights into the teaching of basic math facts including a multitude of instructional strategies, teacher tips, and classroom activities designed to help students master their facts. The emphasis is on strengthening students' understanding of numbers, patterns, and properties as an essential component of math fact teaching. This book provides valuable resources, insights, and options to help you introduce your students to basic facts, provide reviews to support mastery, or develop interventions for students who have not yet mastered basic facts.

In this book, you will find activities and resources for introducing students to basic math facts. You will find tips for generating student talk about math facts including examples of questions and prompts that direct students' thinking toward big ideas and lead them to insights that will simplify the task of mastering the facts. You will find activities to support varied levels of learners so that you can choose the right activity to extend learning for high-level students or modify skills to support struggling students. You will find strategies that are hands-on, engaging, and interactive to motivate reluctant students. You will find activities perfect for small-group interventions and others that work well for whole-class instruction or individual support. And you will find a CD filled with resources to ease your planning and preparation.

This book is a compilation of strategies and activities that are organized to provide a solid math facts program; however, the individual activities and strategies can be easily integrated into your existing math program to provide you with additional resources and varied instructional approaches. You may read the book from start to finish or you may focus on specific sections that address your needs. Consider your students and select the strategies and activities to match their needs, interests, learning styles, and abilities.

How is this book organized?

Throughout the following chapters, a multitude of teaching strategies and activities are shared to build students' understanding and automaticity with math facts. Each chapter is organized to develop essential understanding and provide a menu of possible activities for instruction, practice, and assessment. Following are highlights of the key elements in Chapters 2 through 8.

Making Connections and Focusing on the Big Ideas

Each chapter begins by connecting the new fact set to students' previous experiences and provides a brief review of big ideas that play a key role in students' understanding of the facts and students' development of strategies related to the facts.

Developing Understanding

Each chapter provides two introductory lessons that focus on developing conceptual understanding of the highlighted math facts. One lesson is a Literature Link, introducing the facts through a story context. The other lesson, Exploring the Facts, provides a language-based and/or hands-on exploration of the new set of facts. The activities in this book employ varied instructional techniques, including the use of manipulatives, visuals, tables, literature, hundred charts, and discussions, ensuring that students experience addition and subtraction facts in diverse ways and that each student will be likely to experience these facts in a way that makes sense to him.

In Supporting All Learners, you will find more ideas for those students who may need additional or different types of experiences to develop understanding of the targeted facts. These activities might be done with the whole class but may also be perfect for small teacher-led groups of students. For some sets of facts, you may choose to use several of these activities; at other times, your students may not need the additional exposure. These activities simply provide you with more and varied possibilities for developing understanding.

Building Automaticity

This section focuses on building students' fluency and is broken into two parts: Targeted Practice and Monitoring Progress. In Targeted Practice, a variety of activities are shared that provide practice for that specific set of math facts. Students will have fun rolling number cubes, spinning spinners, and pulling number cards from a deck as they engage in ongoing practice through interactive

activities. It is through repeated and targeted practice that students gain fluency with math facts. Templates for these activities can be found on the accompanying CD.

Along with repeated practice to gain fluency with math facts, students need constant monitoring to ensure that they are progressing in their mastery of facts. Monitoring Progress provides ideas for monitoring students' progress toward automaticity including ideas for conducting frequent fact checks, techniques for tracking students' progress, and suggestions for varied ways to monitor progress including student conferences, progress graphs, and individual goal setting.

Connecting to Subtraction

Addition facts are the primary emphasis throughout this book because of our focus on building math fact fluency. When posed with a subtraction math fact, the most efficient way to solve it is by knowing the related addition fact. When the recall of addition facts is automatic and students understand the connection between addition and subtraction facts, their fluency with subtraction facts naturally increases.

Lessons to develop students' understanding of related subtraction facts are included in each chapter. In addition, you will find suggestions throughout the book of activities to build subtraction fact fluency. To attain fluency with subtraction facts, students need ongoing opportunities to practice the facts and explore their connection to addition facts.

What is the teaching sequence of math facts within this book?

The chapters of this book are organized based on strategies that support students' understanding of addition and subtraction facts. These strategies allow students to make sense of math facts and, therefore, support their mastery of these facts. The strategies are based on big ideas about numbers and operations. The understanding of these big ideas helps students create effective math strategies related to the facts and to ultimately commit those facts to memory. This book is not simply a collection of activities; it is intended to highlight big ideas that provide a perfect focus for math facts instruction, to broaden your repertoire of instructional strategies, to provide you with dozens of easy-to-implement activity ideas, and to stimulate your reflection

related to the teaching of math facts. In reviewing the organization of this book, you may notice that the math facts appear in a sequence that focuses on the complexity of the number concepts and carefully links each new set of facts to previously explored facts, building upon students' prior knowledge.

Rather than asking our students to memorize 121 combinations of addition facts and then 121 combinations of subtraction facts, this book focuses on helping students understand groups of facts and then building on that understanding with additional sets of facts. Over time, our students are given a strong foundation of number sense and number understanding.

Students begin by exploring facts that are one more and two more. These +1/+2 facts are linked to their counting experiences, as they are able to *count on* to find the sums. Exploring facts that are one less and two less allows students to investigate the addition/subtraction connection.

Once students have explored and begun to practice these facts, addition with zero is presented. Although the +0 facts are actually easier for automaticity than +1/+2 facts, the concept of joining is a bit harder to understand (What does it mean to add zero? Why do we add if we have nothing to add?). These facts are better addressed after students have had multiple experiences with +1/+2 facts and have developed the concept of addition.

Ten as an addend is explored next to allow students to develop automatic recall of +10 facts (e.g., 10 + 2 = 12 or 4 + 10 = 14). This skill will be critical later as students use +10 facts as a way of simplifying facts that are near 10 (e.g., 9 + 4 is simplified to 10 + 3).

Exploring doubles is addressed next. Through hands-on experiences, students explore the concept of doubling, find the sums of doubles, and begin to practice these facts for fluency. The automatic recall of doubles facts provides a foundation for more complex facts.

Next, students explore facts with a sum of 10. With 10 being central to our number system, students need many opportunities to explore ways to combine numbers to form 10. Once these facts are mastered, students have developed a strong foundation on which to build mastery of other facts. If students know that 8 + 2 = 10, they can use that understanding to find the sum of 8 + 4 or 8 + 5.

Once our students have explored +1, +2, +0, +10, doubles, and making-ten facts, they have been exposed to the *foundation facts*. The remaining unknown facts can be found by building on this foundation. At this point, students have mastered 83 of the 121 addition facts. And they are armed with an understanding of tens and doubles, which will help them with their still unknown facts.

Using tens is a strategy that assists our students with sums that are near-ten facts. They have not yet explored $8 + 3$ but their knowledge that $8 + 2 = 10$ aids them. "It's 11 because 3 is just 1 more than 2 and $8 + 2 = 10$, so $8 + 3 = 11$!"

Next, our students explore more unknown facts using their doubles knowledge. Their previous experiences with doubles facts support students with more difficult facts like $6 + 7$. "I know $6 + 6 = 12$, so $6 + 7$ is just 1 more. It's 13!"

Through their known facts and their previously explored strategies, students now build mastery with the remaining facts. Although most of the basic facts have been connected to a specific strategy, it is important that students recognize the flexibility of these strategies, knowing that several strategies may work for a given fact. This flexibility allows students to find the answers for the two math facts that have not been specifically addressed (e.g., $5 + 3$, $6 + 3$). Although flexibility of strategies is addressed throughout the program, the final pieces of the teaching sequence focus on different ways students might use known facts to find an unknown fact. Discussions that show flexibility are critical to expanding students' thinking about numbers and the many ways they can be joined or separated. These students share their thoughts about finding the sum of $5 + 3$:

"5 + 2 = 7 and it's just 1 more, so 5 + 3 = 8."
"If you double 3 it's 6, and 5 is 2 more than 3, so 5 + 3 = 8."
"7 + 3 = 10, so 5 + 3 = 8 because it's 2 less than 10."

There is more than one way to think about joining numbers. These comments demonstrate students' strategic reasoning and their enhanced understanding of numbers.

Connecting new facts to previously discussed number concepts allows students to continually build mastery of addition and subtraction basic facts. Figure 1 outlines a brief rationale for the sequence in which the facts are introduced within this book. We recognize, however, that students and instructional programs differ and that teachers might choose, or be required, to introduce facts in a different sequence. Although we believe that there is strong justification for this sequence, we have carefully developed strategies and activities that support instruction of math facts even if the order in which you present the facts differs from the sequence described below.

The lessons and activities in this book focus on strengthening students' number concepts to support their mastery of basic math facts. Teachers who have a deep understanding of big ideas related to numbers and the ways in

Foundation Facts	
+1/+2	Students build on their understanding of counting by exploring 1 or 2 more and 1 or 2 less.
+0	Using their knowledge of the concept of addition, students explore what happens when they add or subtract nothing from a quantity.
+10	Adding 10 to a single-digit number results in a 2-digit sum. Students explore adding 10 in order to build understanding and automaticity that will be needed later when exploring the using-ten strategy.
Doubles	Students explore the concept of doubling and what it means to add 2 groups of equal size.
Making Ten	Because 10 is foundational in our number system, students explore the different ways in which 2 addends result in a sum of 10. This knowledge becomes critical as they later explore using tens to find unknown facts.
Building on the Foundation	
Using tens	Now that students know combinations of addends that have a sum of 10, they use their understanding of the flexibility of numbers to find ways to break apart addends to create simpler facts by using tens (e.g., 9 + 7 is changed to 10 + 6).
Using doubles	Students' knowledge of doubles facts is now put to use to find unknown facts that are near-doubles (e.g., 4 + 5 might be thought of as 4 + 4 + 1).

Figure 1. *This suggested teaching sequence begins with simpler facts and then connects each new set of facts to students' previous experiences.*

which those big ideas relate to the teaching of math facts, and who have developed a repertoire of instructional techniques and classroom activities to highlight those big ideas, are able to simplify the task of mastering basic math facts for their students.

Why are activities and resources on a CD?

Along with the many easy-to-implement student activities described within the book, you will find a teacher-friendly CD filled with customizable versions of the activities. Because the CD materials are produced in a Microsoft Word format, you can easily modify the activity pages to make them simpler or more complex, personalize the tasks to insert your students' names or names of familiar places or events, and adapt the activities to work with any set of math facts. On the CD, you will find the featured activities (described within the book) for each set of facts as well as some additional activities, often modified from a featured activity for a different set of facts, to provide a multitude of practice tasks from which you can choose. The CD also includes teacher

tools (e.g., hundred charts, addition tables, assessments, and game templates) to simplify your planning and reduce your preparation time.

Resource for Professional Learning Communities

Effective teachers constantly reflect on their own teaching. They gather new ideas, try them with students, reflect on their successes, and find ways to continually refine their teaching. At the conclusion of this book, questions are posed to stimulate reflection about the key points within the chapters. These guiding questions are designed for your personal reflection or for use in school-based study groups. Discussion about math facts instruction within our professional learning communities broadens our understanding and improves our teaching.

Our Goal

The purpose of this book is to explore ways to support all students in mastering addition and subtraction facts. By focusing on big ideas, strengthening students' understanding of math operations, developing strategic thinking, and providing varied and engaging practice tasks to promote fluency, our students will be better equipped to both understand math facts and commit the facts to memory. Whether you are introducing students to basic facts, reviewing facts, or providing remediation for struggling students, this book provides you with insights and activities to simplify this complex and vital component of math teaching.

Doubles

+	0	1	2	3	4	5	6	7	8	9	10
0	**0**	1	2	3	4	5	6	7	8	9	10
1	1	**2**	3	4	5	6	7	8	9	10	11
2	2	3	**4**	5	6	7	8	9	10	11	12
3	3	4	5	**6**	7	8	9	10	11	12	13
4	4	5	6	7	**8**	9	10	11	12	13	14
5	5	6	7	8	9	**10**	11	12	13	14	15
6	6	7	8	9	10	11	**12**	13	14	15	16
7	7	8	9	10	11	12	13	**14**	15	16	17
8	8	9	10	11	12	13	14	15	**16**	17	18
9	9	10	11	12	13	14	15	16	17	**18**	19
10	10	11	12	13	14	15	16	17	18	19	**20**

KNOWN FACTS | **Doubles** TARGETED FACTS | UNKNOWN FACTS

We continue to add to our students' repertoire of math facts, this time focusing on doubles facts. Because both addends are the same, there is no need to address the commutative property, so this fact set is quite small—just eleven doubles facts in all. And because students have already explored and practiced +0, +1, +2, and +10 facts, they already know the sums for 0 + 0, 1 + 1, 2 + 2, and 10 + 10, leaving only seven unknown doubles facts. These facts are usually easy for students to recall and provide important links to future facts. Students will later refer back to these facts to find the sums of more complex near-doubles facts.

Focusing on the Big Ideas

Exploring big ideas about mathematics provides the backdrop for our exploration of doubles facts. Some big ideas include the following.

Doubling is the process of joining two groups of the same quantity.

The term *doubling* refers to the process of adding a quantity to itself. Doubling is joining two *like* groups. It is when both parts that make up a whole are equal quantities.

Halving is the opposite of doubling.

Separating a set into two equal groups results in halves. By halving a set, we can determine the quantity for the two equal parts that make the whole. Separating half from a set will result in a difference that is the same as the amount that was separated (i.e., in $12 - 6 = 6$, 6 was separated from the set and 6 remain in the set).

Addition and subtraction are inverse operations.

If students know that doubling 5 results in 10, or $5 + 5 = 10$, then they will also know that halving 10 results in 5, or $10 - 5 = 5$. Knowing a doubles fact (addition) supports students in knowing halves (subtraction).

Key questions related to the big ideas for doubles are:

What does it mean to double an amount?
What does it mean to find half of an amount?
How are doubling and halving alike? How are they different?

Our goal is to continually reinforce the big ideas related to math facts as we help students develop addition and subtraction strategies.

Understanding Doubles

Literature Link: *Double the Ducks*

In *Double the Ducks* (2003), Stuart J. Murphy tells the story of a boy with 5 ducks. When 5 duck friends follow them home, the boy must double everything as he cares for the 10 ducks.

Before Reading Ask students to brainstorm types of animals that live on a farm. Ask them what kinds of things a farmer does to take care of the different animals. Ask students what it means to double something. Can they give an example of a double? Read the title and ask students to predict what the story might be about.

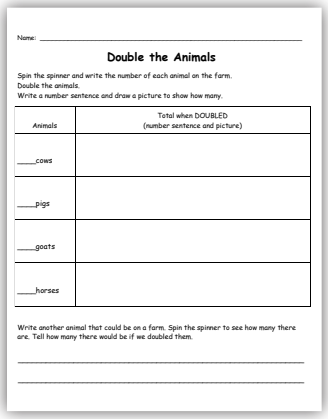
During Reading As you read each number in the story and what it represents, record the number and item on chart paper (e.g., 1 person, 2 hands, 3 sacks of food, etc.). Pause after reading that each duck brought back a friend. Ask students to predict what the farmer will need in order to take care of 10 ducks. Continue reading to check their predictions.

After Reading After finishing the story, compare the predictions that students made with the actual events of the story. Talk about what it means to double by referring to the items on the chart paper as examples. Pose a few related farm problems, having students turn and share the double with a partner. Keep the quantities small to assess students' understanding of the concept of doubling rather than their computation skills.

His ducks love to eat bread. He needs 4 loaves of bread to feed his ducks. How many loaves of bread will he need if we double the ducks? How do you know?

His pigs are very muddy. He uses 3 bars of soap to bathe them. How many bars of soap will he use if we double the pigs? How do you know?

His horses love to eat sugar cubes. He needs 5 bags of sugar cubes for his horses. How many bags of sugar cubes will he need if we double the horses? How do you know?



Name: _____

Double the Animals

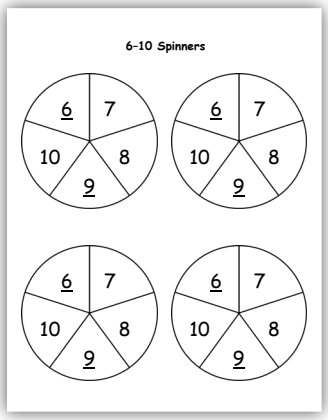
Spin the spinner and write the number of each animal on the farm.
Double the animals.
Write a number sentence and draw a picture to show how many.

Animals	Total when DOUBLED (number sentence and picture)
____cows	
____pigs	
____goats	
____horses	

Write another animal that could be on a farm. Spin the spinner to see how many there are. Tell how many there would be if we doubled them.

_____

_____



6–10 Spinners

Once you are confident that students understand the concept of doubling, begin a task to explore doubles with addends of 6–10. Give students a *Double the Animals* recording sheet (see CD) and a set of 20 counters to represent animals. Students will work with partners to find the doubles sum for a set of cows, pigs, goats, and horses. Have partners work together, with one partner spinning a 6–10 spinner (see CD) to see how many cows are on the farm, and the other partner placing counters in a row to represent that number of cows. Partners then double the cows by creating a similar row to show the doubles set. Partners find the total number of cows, the sum, and record the addition number sentence on their recording sheets. Have partners switch roles

and spin again, finding the number of pigs and the double for that number of pigs. As they work together to double all of the animals, observe how they are finding the sums. Do they know what it means to double? Are they counting all of the counters? Are they using any strategies? Do they have automatic recall of any doubles facts? Make a note of students who already have recall of some doubles facts. For the next lesson, those students might be transitioned to some of the doubles practice tasks, and those who were noted to be struggling with the concept might be asked to join you for additional explorations and discussions of the doubling process.

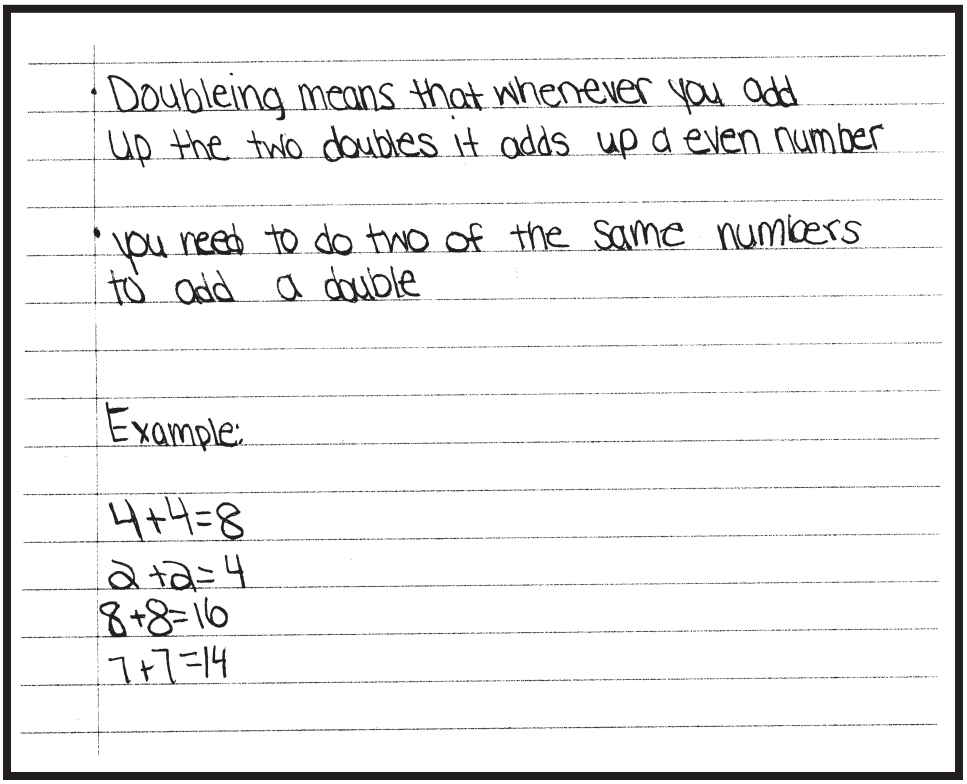
Once pairs have determined doubles for their animals, have them share some of their doubles facts, recording the number sentences on the board to provide a check for accuracy of the 6–10 doubles facts.

Finally, have students respond to the following prompt, as in Figure 5.1, in their math journals.

What does it mean to double a number?

If students have a difficult time explaining their understanding of doubles in words, remind them that they can draw pictures or give examples to show

Figure 5.1 *This student shares her understanding of doubling.*



their thinking. Sharing their writing with a partner, or presenting it during circle time, is an effective way to summarize the doubling concept.

Exploring the Facts: Creating Equal Sets

Visualizing the doubling process helps students better understand its connection to an addition equation. When considering the part-part-whole concept, both parts are of equal size. Model equal sets for students by creating sets on an overhead projector, document camera, or SMART Board. Lining objects in two equal rows helps students quickly see that sets are equal.

Pose some doubles problems.

4 orange fish were in the fish bowl. 4 black fish were in the fish bowl. How many fish were in the fish bowl?

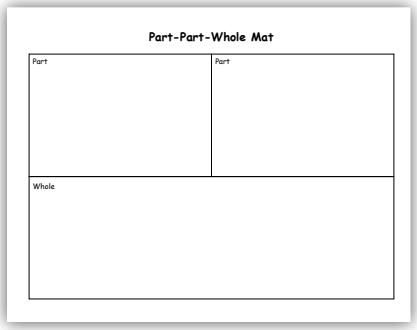
6 boys were at the park. 6 girls were at the park. How many children were at the park?

Bailey had 7 flower stickers. She had 7 bunny stickers. How many stickers did she have?

Liam had 6 red cars and 6 blue cars. How many cars did he have?

Colin had 9 soccer balls. He had 9 baseballs. How many balls did he have?

Allow students to use manipulatives to create the two equal-sized groups. A part-part-whole mat (see CD) provides a great template for this exploration. Students place a group of manipulatives in the top left section, then create an equal-sized group in the top right section. Pause to ask students about the two groups. Are they the same size? Have them record an addition equation to show the two parts, and then pull the two parts together to make one whole group. Have them find and record the sum. Continue to pose problems as you observe students at work.



Tip The use of motion, pulling groups together, provides an important visualization of the concept of joining, or finding a total or whole set. An overhead projector or document camera allows us to physically move sets together, as do many SMART Boards. Magnetic graphics or student actors can also be joined to demonstrate the action of addition.

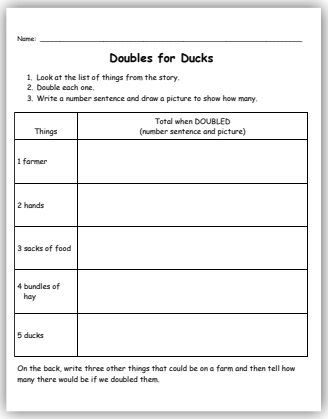
Supporting All Learners

The following classroom activities provide you with additional lesson ideas for the whole class or for small teacher-led groups with students who might need to explore doubles in a different way.

Act Out Doubles Pose doubles problems for students to act out. You might have 4 students put their hands on their heads and then double the number of students putting their hands on their heads. You might have 6 students jump up and down and then double the number jumping up and down. Have fun doubling the students and recording the addition number sentences to represent the doubles.

Roll and Double It Provide students with 20 counters. Roll a 1–6 number cube and ask students to make a row with that number of counters. Ask them to double it by making another row of the same size. Have them count to find the sum. Record the addition number sentence. Roll another number and repeat the process.

Ask students how they will know they are doubling when they see an addition number sentence. Do they notice that both addends in the number sentence are the same? Do they understand that means the size of each group is the same? Ask questions to guide students in observing the connection between the numbers in the equation (the same numbers) and the number of counters in each group (the same size groups).



Name: _____

Doubles for Ducks

1. Look at the list of things from the story.
2. Double each one.
3. Write a number sentence and draw a picture to show how many.

Things	Total when DOUBLED (number sentence and picture)
1 farmer	
2 hands	
3 sacks of food	
4 bundles of hay	
5 ducks	

On the back, write three other things that could be on a farm and then tell how many there would be if we doubled them.

Revisit *Double the Ducks* For struggling students, it may be helpful to reread *Double the Ducks* and allow them to use manipulatives to double each item in the story. Students can use the *Doubles for Ducks* recording sheet (see CD) to show their work. Watch them as they work, asking questions to guide their thinking and assess their understanding. After completing the story items, brainstorm other farm items and have them select a few and find the doubles.

Magical Doubles Share this doubles story to engage students in solving and writing doubles problems.

Mork was a famous magician. Children loved to see his magic shows. Mork made a penny disappear and then reappear behind someone's ear. Amazing! He cut a scarf in half and with a wave of his wand made it a whole scarf again. Unbelievable! But most of all, children loved when Mork took out his magic hat. Mork put 1 cuddly bunny into his hat, waved his wand and chanted: "Magic hat, don't give me trouble. These children want to see a double!" He reached into the hat and pulled out 2 cuddly bunnies! The children cheered and shouted for more. Mork reached into his pocket and pulled out 2 shiny quarters, put them into his hat, waved his magic wand, and chanted: "Magic hat, don't give me trouble. These children want to see a double!" He reached into his hat and pulled out 4 shiny quarters!

Ask students to turn and tell a partner what Mork pulled out of his magic hat when he put these items into it:

3 purple purses
4 red balls
5 salty pretzels
6 juicy grapes
7 silver spoons
8 spotted owls
9 rotten potatoes
10 gray mice

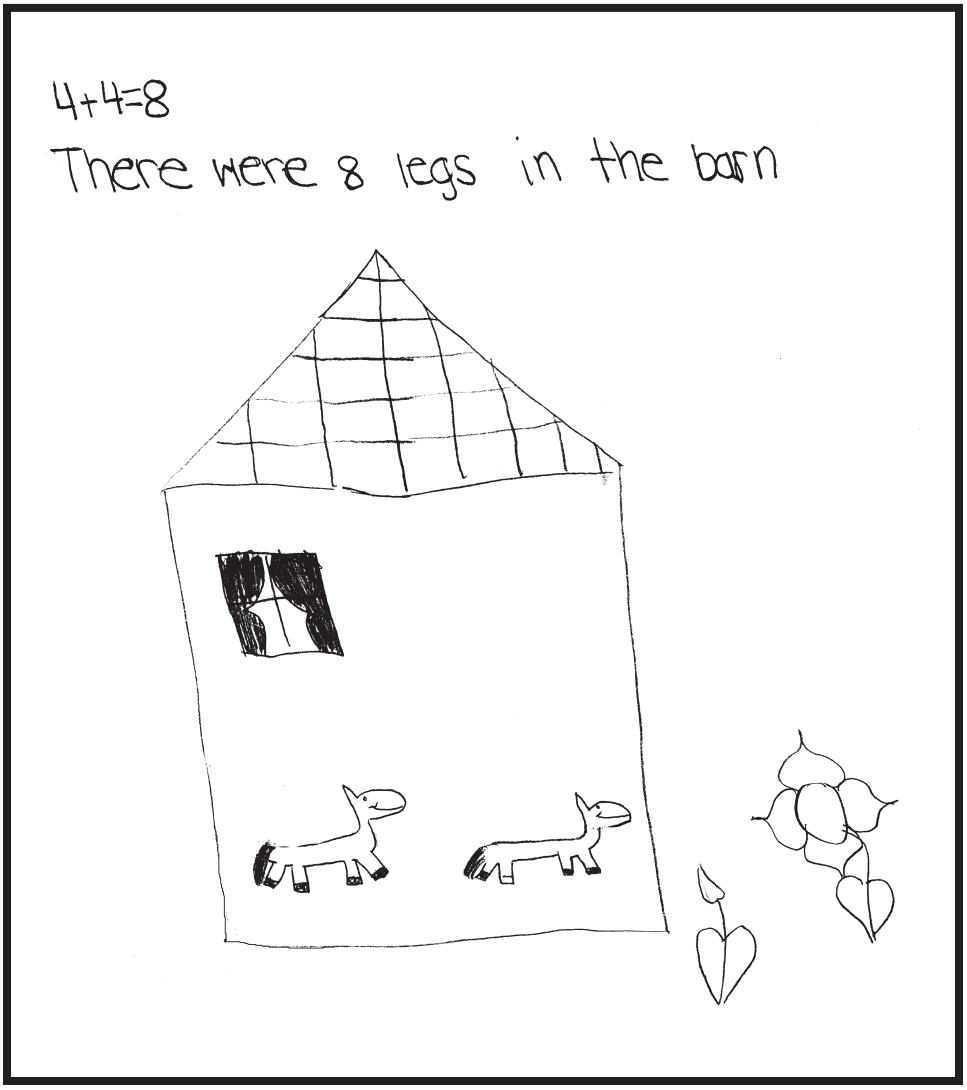
Allow students to use beans or counters to explore the sums, if needed. Record each doubles number sentence on the board.

Have students talk with partners to decide on something else to put in Mork's hat. Go around the room to have pairs tell you what Mork put in and what he took out.

Class Doubles Book As a class, make a list of things that come in 1, 2, 3, 4, and so on (i.e., 1 trunk on an elephant, 2 horns on a bull, 3 lights on a traffic light, 4 legs on a table, 5 fingers on a hand, etc.). Have students pick an item from the list and figure out how many there would be if the items were doubled. Have each student draw a picture and write a number sentence for a class doubles book (see Figure 5.2).

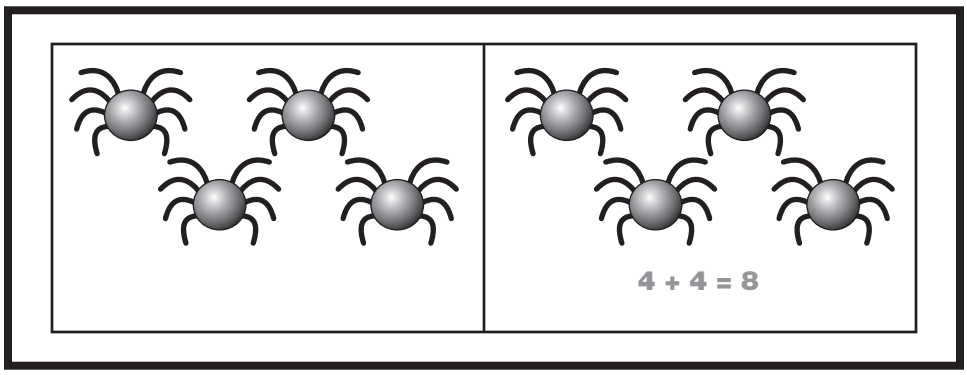
Holey Doubles Give each student a piece of paper and have them fold it in half. Students pick a 1–10 number card (see CD) and, with the paper folded,

Figure 5.2 *This page for the class doubles book describes the legs on 2 horses.*



use a handheld paper punch to punch that number of holes in their paper. Students then open the folded paper to reveal double the holes. Students record the doubles number sentence on their paper and write a doubles story about their holes (i.e., "There were 5 spiders and 5 more crawled in. How many were there?"). Students can get creative and decorate around the holes to make them look like flowers or spiders or lollipops or suns. The doubles art can be posted on a bulletin board to display a variety of doubles equations as in Figure 5.3.

Figure 5.3 *Four holes are punched on the outside of the folded paper to reveal double the holes when the paper is opened. Students can decorate the holes to look like spiders and add a doubles number sentence.*

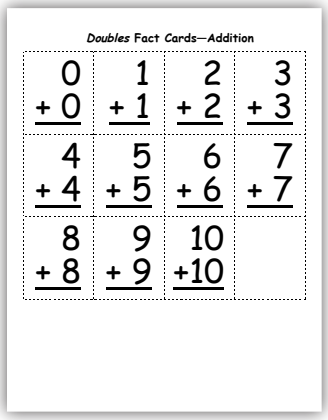


Building Automaticity

Targeted Practice

Individualized Fact Card Practice Referring to each student's Fact Check will allow you to select specific fact cards that meet her needs. Are there specific doubles facts that this student struggles to recall? Practice with targeted facts allows for more repetition of those facts and narrows the scope of the task for each student. Students can be given the job of finding their "focus facts" in their deck of fact cards before beginning a focused fact card practice.

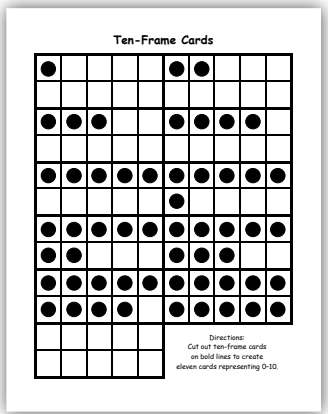
A Fact Card Menu Fact cards (see CD) can be a source of activities that students do independently at school or at home. The following activities alleviate the stress that is sometimes associated with fact cards as they emphasize knowledge of the answer rather than speed. Suggest one of the following for students who need additional practice with the facts.

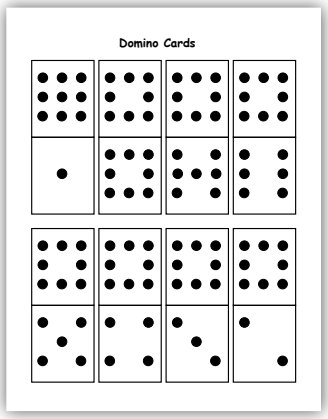


- Pick a card and draw a picture to show the fact.
- Pick an addition fact card and write a subtraction fact that goes with it.
- Pick a card and write the fact three times.
- Pick a card and write a story problem for the fact.
- Pick a card and show the fact on a number line. Record the number sentence by the number line.

The Teacher's Role During Game Time While students play games, you might either observe their play or capitalize on your opportunity to work with a small group of students. Watching students as they play games allows you to gain insights about their fluency with math facts. You will quickly identify those who have mastered facts and those who need additional support.

Although those observations are extremely helpful, it might also be beneficial to take the opportunity to address the needs of specific students. As most students are developing their rapid recall of facts through the games, you might conduct small-group assessments, lead intervention groups, or conduct individual interviews. Think about both assessment and instruction as you balance opportunities to observe students at play and support students individually or in small groups.





Double Ten-Frames Provide each pair with two sets of ten-frame cards (see CD) containing frames with 0–10 dots. Have students work with partners to find doubles (i.e., locate the frames with 6 dots on each frame), find the sums, and record the number sentences.

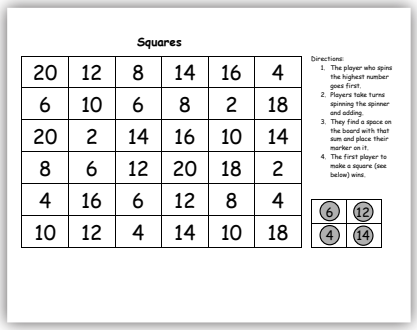
I Spy Doubles Have students search for double dominoes, dominoes with the same number of dots on each side, and write matching addition equations (see Figure 5.4). See the CD for domino templates.

Figure 5.4 *These students search for double dominoes.*

1	9	5
4	3	6
7	8	2

Figure 5.5 *Tossing beanbags at a number chart on the floor, and doubling the number the bag lands on, incorporates movement into math facts practice.*

Beanbag Addition Create a floor mat with a white shower curtain liner or large poster board, marking nine sections, each containing a number from 1–9 (see Figure 5.5). Students take turns tossing a beanbag and calling out the number it lands on and the double. Students record their double equations.

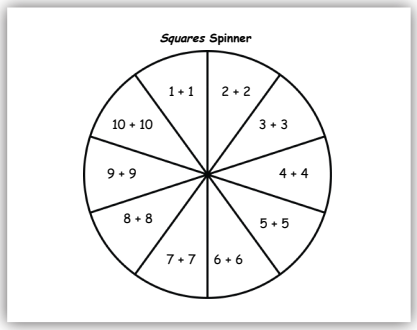

Squares

20	12	8	14	16	4
6	10	6	8	2	18
20	2	14	16	10	14
8	6	12	20	18	2
4	16	6	12	8	4
10	12	4	14	10	18

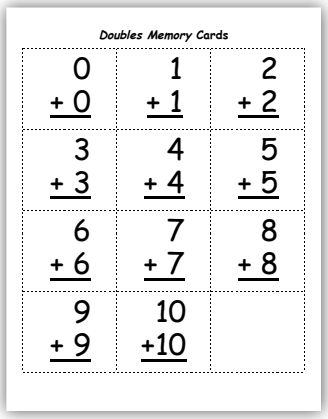
Directions:
1. The player who spins the highest number goes first.
2. Players take turns spinning the spinner and adding.
3. They find a space on the board with that sum and place their marker on it.
4. The first player to make a square (see below) wins.

6 12
4 14

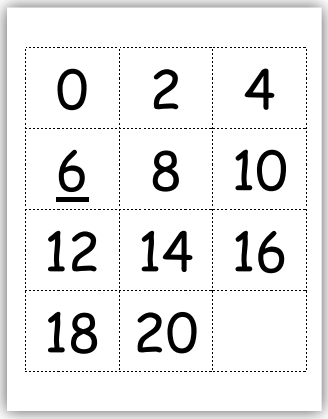
Squares The goal of *Squares* is to be the first player to have markers on four numbers that form a square on the game board. Students play with partners. Each pair needs a *Squares* spinner and game board (see CD) along with game markers (e.g., beans or counters). Players take turns spinning, finding the sum, and placing their marker on the board. The first player to have four markers arranged to form a square is the winner. For a noncompetitive option, have partners work together to spin, add, and cover doubles sums, talking together about where to place each marker as they try to form a square. When a square is formed, students clear the board and start again.



Doubles Memory In *Doubles Memory*, students try to match a doubles fact card (5 + 5) with the correct sum (10). Students work in pairs and spread the cards (see CD) facedown on a desk, table, or floor. Each player takes turns choosing two cards. If the cards go together (the doubles fact and the sum), the player keeps the cards and gets another turn. If the cards do not match, the player returns them to the same spot on the desk, facedown, and his turn is over. The player with the most cards at the end of the game is the winner.


Doubles Memory Cards

0 + 0	1 + 1	2 + 2
3 + 3	4 + 4	5 + 5
6 + 6	7 + 7	8 + 8
9 + 9	10 + 10	



0	2	4
6	8	10
12	14	16
18	20	

Monitoring Progress

Individual Teacher-Guided Fact Checks Individual student assessment might be indicated for students who have extreme difficulty with the cumulative Fact Checks. Using a set of fact cards, assess the child's fluency verbally, beginning with simpler facts and then moving to those that are causing difficulty. As a child misses, set the card aside. When a child misses five facts, stop and set a goal for those facts. Give the missed fact cards to the student, discuss any strategies that might help them better remember those facts, and determine a time frame for when you will recheck for mastery of the facts.

Tip Many students have difficulty with timed tests. Teacher-administered oral fact checks provide a clear picture of fact mastery.

Connecting to Subtraction

As students develop an understanding of doubles facts, take every opportunity to talk about the connection between addition and subtraction facts. Discussing halves will demonstrate the inverse of doubles and show what happens when one half is separated from the other—the same amount always remains. The two addends in a doubles number sentence can be thought of as halves. Thinking *halves* simplifies finding the difference for doubles subtraction.

Literature Link: *Martha Blah Blah*

In *Martha Blah Blah* (1996) by Susan Meddaugh, a dog gains the ability to speak when eating alphabet soup but encounters problems when half of the letters are removed from the soup.

Before Reading Have students turn and tell a partner what they think dogs might say if they could speak. Share a few of their ideas. Tell students you will be reading a story about a dog who speaks. Before you begin reading, have students recite the alphabet as you record the letters on chart paper or the board. Count the total number of letters in the alphabet and record *26* on the board.

During Reading After reading that Granny Flo removed half of the letters, pause to identify and cross off the letters she removed. Ask students to predict if she removed more than, less than, or about half of the letters.

After Reading Ask students if the 13 letters removed were half of the alphabet. Give partners 26 counters to represent the letters. Have students explain how they know that 13 is half of 26.

Prepare bowls of cereal letters, or magnetic letters, to represent bowls of alphabet soup. Have bowls with the following amounts of letters: 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20. The actual letters in each bowl do not matter, just the quantity of letters in the bowls. Have students work in pairs or groups to count the letters in their bowls and then remove half of the letters like Granny Flo did. Students must find out how many letters are left in their bowls and record the subtraction number sentence on the *Letters in Our Soup* recording sheet (see CD) as in Figure 5.6. As students finish, have them switch bowls with another group and do it several more times.

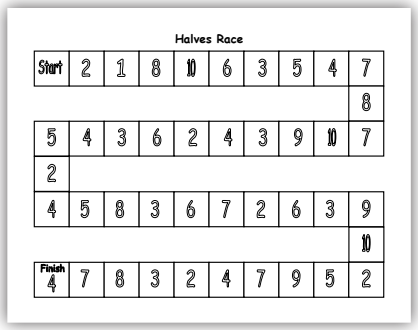
After students have worked with several bowls of soup, have students share, as you record, the different number sentences for half of the letters. Once you have completed the subtraction number sentences, write associated doubles addition sentences next to the subtraction sentences. Ask students to describe how the addition and subtraction sentences are the same and how they are different. Their comments will likely include:

They have the same numbers in them, but in different places.
They all have 2 numbers that are the same and 1 that is different.
If you take a half away, you get the other half.

Figure 5.6 *This student explores subtraction by removing half of the letters from his soup.*

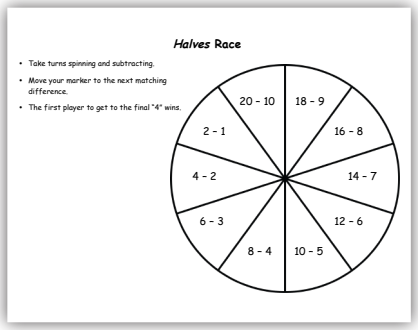
Letters in Our Soup

Letters in the bowl	How many would be left if we took half away?	Number sentence
14	7	14 − 7 = 7 7 letters Left
10	5	10 − 5 = 5 5 letters left
16	8	16 − 8 = 8 8 letters left



Halves Race

Start	2	1	8	10	6	3	5	4	7
									8
5	4	3	6	2	4	3	9	10	7
2									
4	5	8	3	6	7	2	6	3	9
									10
Finish 4	7	8	3	2	4	7	9	5	2



Halves Race

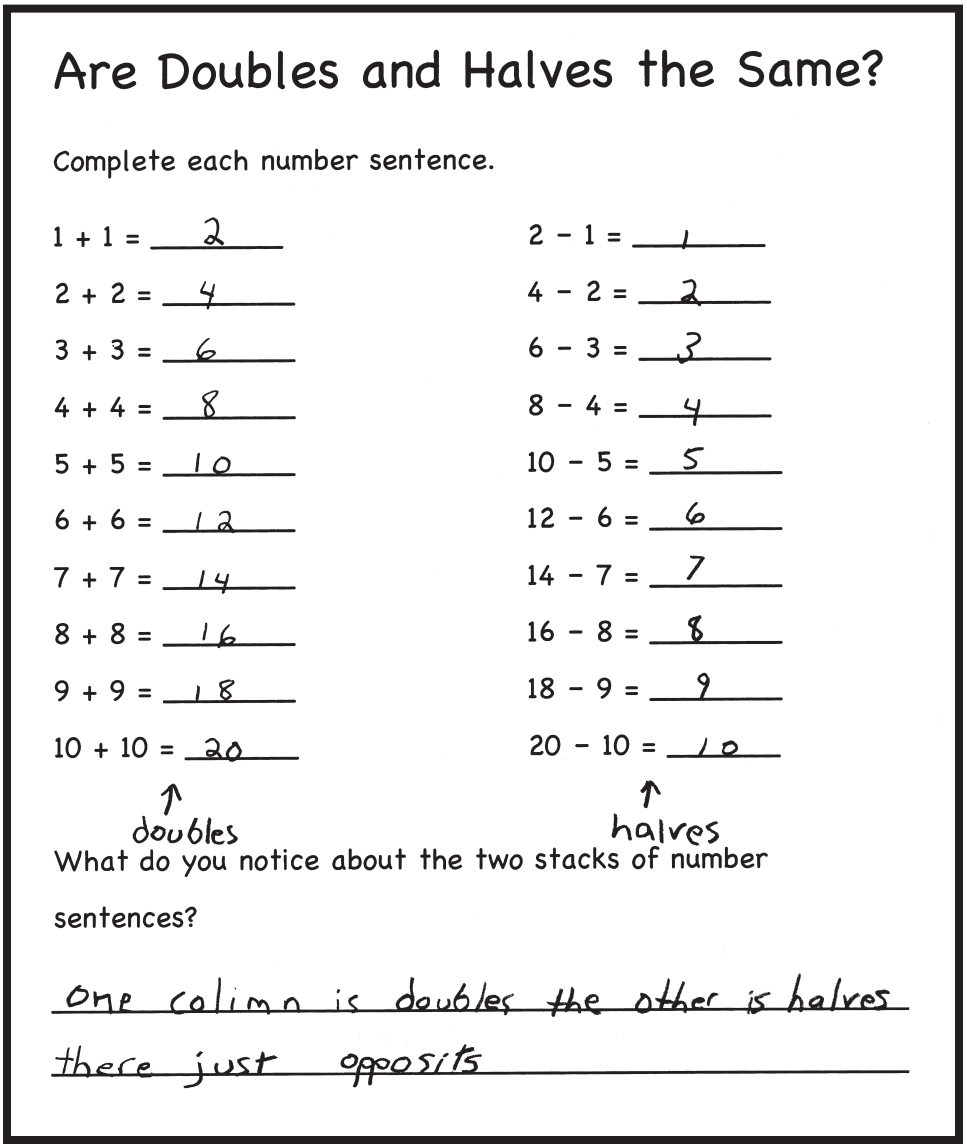
- Take turns spinning and subtracting.
- Move your marker to the next matching difference.
- The first player to get to the final "4" wins.

Or students might use the *Are Doubles and Halves the Same?* recording sheet on the CD to explore the similarities and differences between addition and subtraction doubles facts as in Figure 5.7.

Practicing for Fluency

Halves Race In *Halves Race*, students spin and subtract in order to move along the game board. Each pair need one *Halves Race* game board, a doubles subtraction spinner (see CD), and game markers. Students take turns spinning, finding the difference, and moving to the next space containing that difference on the board. The first player to the finish wins.

Figure 5.7 *Students gain insights when they are asked to observe math facts.*



Are Doubles and Halves the Same?

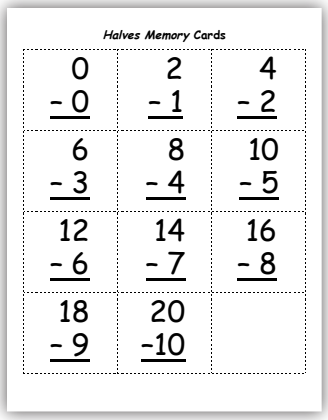
Complete each number sentence.

1 + 1 = 2	2 − 1 = 1
2 + 2 = 4	4 − 2 = 2
3 + 3 = 6	6 − 3 = 3
4 + 4 = 8	8 − 4 = 4
5 + 5 = 10	10 − 5 = 5
6 + 6 = 12	12 − 6 = 6
7 + 7 = 14	14 − 7 = 7
8 + 8 = 16	16 − 8 = 8
9 + 9 = 18	18 − 9 = 9
10 + 10 = 20	20 − 10 = 10
↑ doubles	↑ halves

What do you notice about the two stacks of number sentences?

one colimn is doubles the other is halves there just opposits

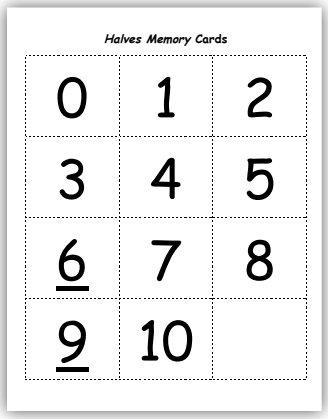
Halves Memory In *Halves Memory*, students try to match a doubles subtraction fact card (10 – 5) with the correct difference (5). Students work in pairs and spread the cards (see CD) facedown on a desk, table, or floor. Each player takes turns choosing two cards. If the cards go together (the subtraction fact and the difference), the player keeps the cards and gets another turn. If the cards do not match, the player returns them to the same spot on the desk, facedown, and her turn is over. The player with the most cards at the end of the game is the winner.



Halves Memory Cards

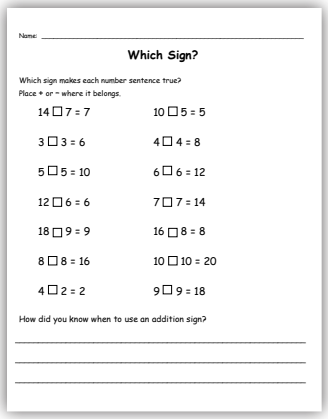
0 - 0	2 - 1	4 - 2
6 - 3	8 - 4	10 - 5
12 - 6	14 - 7	16 - 8
18 - 9	20 -10	

Which Sign? *Which Sign?* provides a review of doubles addition and subtraction facts and builds foundational algebra skills as students explore building equations. Partners work to complete the *Which Sign?* recording sheet (see CD), deciding if an addition or subtraction sign would make the equation true (i.e., Would + or – complete the number sentences 6 ☐ 6 = 12 or 16 ☐ 8 = 8?). Rather than making copies of the worksheet for each student, consider putting it in a plastic sleeve and placing it at a center. Students then copy each equation on a blank paper, inserting the sign they choose. Asking students to write about how they made their decision gives insight into their thinking. How did they know where to place an addition sign or subtraction sign?



Halves Memory Cards

0	1	2
3	4	5
6	7	8
9	10	



Name: _____

Which Sign?

Which sign makes each number sentence true?
Place + or – where it belongs.

14 ☐ 7 = 7	10 ☐ 5 = 5
3 ☐ 3 = 6	4 ☐ 4 = 8
5 ☐ 5 = 10	6 ☐ 6 = 12
12 ☐ 6 = 6	7 ☐ 7 = 14
18 ☐ 9 = 9	16 ☐ 8 = 8
8 ☐ 8 = 16	10 ☐ 10 = 20
4 ☐ 2 = 2	9 ☐ 9 = 18

How did you know when to use an addition sign?

_____
_____
_____

Professional Learning Communities Study Guide

Learning is inherently social. As teachers, it is easy to feel isolated. It becomes especially important that we find opportunities to talk with colleagues and reflect on our teaching practices. Through professional learning communities, we hear new ideas, consider new techniques, clarify our thinking, and ultimately enhance our teaching. It is through conversations with colleagues that we grow as teachers. Teacher study groups value the experience and knowledge of teachers. They provide a forum for rich discussions about teaching and learning. They motivate us to try new approaches and assess our own practices.

Although there are many ways to structure a study group, it is most important to foster a climate in which teachers feel free and safe to participate in the ongoing conversations and exchange of ideas. These study groups should be designed with your teachers in mind. They should focus on the needs of your students and fit the culture of your school. Whether you meet once a week or less often; whether you focus your meetings on a professional book, student work samples, video clips, or a critical question—make the leap into conversation. Here are a few tips to consider as you plan for implementing a study group.

Consider Group Size Small groups are ideal for study groups, but full-faculty study groups are doable if small-group breakout sessions are an integral part of your planning. You may want to kick off discussion with a general question and then break into smaller groups. Often the optimal number is four to six teachers to ensure there is time for all to exchange ideas. The larger group can reassemble at the end of the session to debrief.

Use Study Questions Starting with a few questions can jump-start your discussions. There are various ways to use questions.

- Put three or four questions in an envelope and randomly pull them out for discussion.
- Create a chart with two or three starter questions and ask the group to generate more, tapping their own interests and needs.

- Create a list of three or four questions and have teachers prioritize the questions based on the needs of their students.
- Decide on three or four questions and divide the group by interest in the various topics. This allows for a more in-depth study.
- Make copies of the suggested questions for everyone and invite discussion without deciding where to start.

Create an Agenda Make sure you have planned a beginning and ending time and *always* honor those times. Teachers are busy and knowing there will be a time to start and a time to end is important. Send the agenda to participants prior to the meeting to remind them of the topics to be discussed, as well as any reading to be completed.

Stay Focused on the Topic State the topic and goals of the session at the start. Plan a procedure that is transparent. You might start by saying something like "Let's decide on a signal to use when we feel the discussion is drifting and then have everyone agree to help stay focused."

Create a List of Norms Simple expectations that are determined by the group often make study groups function with greater ease and increase potential for success. These can be simple and might include ways to invite a tentative member into the conversation, expectations about listening and sharing, start and stop times, and a procedure for refocusing.

Make It Personal Make the learning personal for each participant. You might begin each session with teachers turning to a colleague and sharing a quote or teaching idea that resonated with them.

Share Leadership Rotate group facilitation. Identify several "duties" for the facilitator. Examples might include identifying a question to stimulate discussion, suggesting a big idea from a chapter or group of chapters, posing reflective questions (e.g., "Why do you think the authors kept emphasizing that point?"), and summarizing at the end of the session. Remember that in a study group, *everyone* is a learner. This isn't the place for an "expert"!

Include Everyone Keep groups small enough so that even the quietest member is encouraged to speak. Active listening on everyone's part will help. Remember that periods of silence should be expected when people are thinking.

Encourage Implementation Decide on an activity or teaching technique that participants will try with students between sessions. Having tried some of the ideas allows teachers to bring insights to the next meeting and ensures that the study group goes beyond talk and into action.

Engage in Reflection Stop from time to time to reflect on what you are learning and how you might make your group's interactions more productive. Make sure you take time to enjoy one another and celebrate your learning.

Set Dates for the Next Meeting Always leave knowing when you will meet again, who will facilitate, and what the general focus will be for the upcoming session.

The following questions relate to the content in each chapter. These are suggestions. Many more concepts and ideas are presented in each chapter. Enjoy!

Guiding Questions

Introduction

1. Why is mastery of math facts important? What problems have you observed when students do not know basic math facts?
2. In what ways would a strong understanding of numbers support students as they focus on math facts?
3. What have you observed about anxiety related to memorizing math facts? Are there types of math fact practice activities that increase anxiety or decrease anxiety?
4. What types of activities might motivate students to practice math facts?
5. How might attention to the sequence in which facts are introduced support mastery of the facts?

Chapter One: Understanding Addition and Subtraction

1. Why is it important for students to understand the concepts of addition and subtraction?
2. What problems might occur if students are asked to memorize math facts too soon?

3. What types of models might be used to help students visualize addition and subtraction? Why might using a variety of models be helpful for students?
4. What real-world experiences might create an effective context for addition or subtraction problems?

Chapter Two: Plus One, Plus Two

1. What prior knowledge will help students better understand +1/+2 and –1/–2?
2. What tools or models help students visualize +1, +2 and –1, –2?
3. Why does it make sense to address –1 and –2 shortly after +1 and +2? What misunderstandings could occur? How will you address them?
4. Reflect on the importance of students' understanding of the commutative property. In what ways will it support their success?

Chapter Three: Adding Zero

1. Why might addition and subtraction with zero be confusing for some students?
2. What might you do to clarify the concepts of addition and subtraction with zero?
3. In what ways does the integration of children's literature enhance math facts lessons?
4. What rules will you establish for managing math fact games within your classroom?

Chapter Four: Adding Ten

1. Why is adding ten an important foundational skill?
2. In what ways might tens facts be challenging for students?
3. How will you help students recognize their successes?
4. What visual tools might help students better understand +10 facts?
5. Why are so many of the activities within the book focused on partner discussions? What are the benefits? What might you consider to ensure success with partner tasks?

Chapter Five: Doubles

1. How might you assess fluency for students who struggle with written Fact Checks?
2. In what ways will visual experiences help simplify doubles facts? What tools provide effective visuals of doubles?
3. Why is it important for students to explain their thinking as you explore sets of math facts?
4. How might using the terms *double* and *half* support or confuse students? What might you do to help them understand the terms?
5. What are the benefits of modifying familiar math fact games for different sets of facts?
6. How might you differentiate tasks for different levels of learners?

Chapter Six: Making Ten

1. Why is it important to allow students time to explore tens?
2. How will a deep understanding of tens support students with other facts?
3. What games and practice activities might support automaticity?
4. Why is this a good time to work on fluency of the already explored facts?
5. How can you ensure that your students will get repeated practice with math facts?

Chapter Seven: Using Tens

1. How will students' understanding of tens support them with more difficult facts?
2. Which activities would be good choices for math fact centers? Why?
3. What management considerations are important when setting up math facts centers?
4. How will you provide repeated practice for students who have still not mastered past fact sets?
5. What planning considerations will make a Family Math Fact Night most effective?
6. What is the role of language in developing math fact strategies?

Chapter Eight: Using Doubles

1. How will students' understanding of doubles support them with more difficult facts?
2. Give examples of some addition or subtraction math facts for which students might use different strategies. Explain how each strategy makes sense for the fact. Is one more efficient than the other?
3. Beyond supporting mastery of math facts, what are the advantages of focusing on number strategies?
4. When are students transitioned from activities to build understanding to practice for fluency? What understandings must occur prior to fluency practice? Why?
5. How might home practice of math facts be beneficial for students? Are there any drawbacks? What can you do to promote effective home practice activities?

Conclusion

1. What are the most significant ways in which we should rethink the teaching of math facts?
2. The authors contend that automaticity takes time. How do some programs rush students as they learn math facts? How can we ensure that enough time is allowed for students to master facts?
3. Reflect on the teaching sequence of math facts within the book. In what ways might this sequence benefit students versus the more traditional sequence of +0, +1, +2, +3, +4, and so on?
4. The authors suggest that effective math fact teaching "cultivates reflective students who have a greater understanding of numbers and a flexibility of thinking that allows them to understand connections between mathematical ideas." Do you agree or disagree? Why?
5. What tips would you give to a beginning teacher who is deciding how to approach the teaching of math facts?

Additional Study Group Resource

Viewing short video clips of teachers and students in real classroom situations generates discussion, conveys new instructional approaches, and promotes reflection about teaching and learning. For authentic video clips related to math fact teaching, try the following resource:

Bureau of Education and Research. 2009. *Increasing Your Students' Mastery of Addition and Subtraction Math Facts.* Bellevue, WA: Bureau of Education and Research.

Thank you for sampling this resource.

For more information or to purchase, please visit Heinemann by clicking the link below:

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