
2+1=3
2+2=4
2+3=5

Early Mathematics Fluency with CCSSM

By Gabriel T. Matney

To develop second-grade students' confidence and ease, use these three specific types of tasks that align with Common Core State Standards for Mathematics expectations.

What comes to mind when you hear the phrase, "She is fluent in that"? Perhaps you think of a foreign language teacher whose mastery of another language was so profound that you were never quite sure whether his or her inner monologue was in English or the other language. The qualities of someone fluent in another language involve more than simply recalling words quickly. A fluent speaker must have understanding of both the structure of language and the meanings of words. But the most defining characteristic of fluency is the quality of creative expression, the ability to construct new associations and meanings with the language. If we were to apply this analogously to the context of quantitative fluency, we might ask, "What does it mean for a student to be *fluent* in mathematics, and how do we, as teachers of mathematics, help inspire the development of *fluency* in each of our students?"

These are significant questions to consider, because fluency is an important aspect of *Principles and Standards for School Mathematics* (NCTM 2000) and the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010). For students to reach a meaning for mathematical fluency that mirrors the fluency of a foreign language expert, teachers must create learning experiences that produce opportunities for students to make sense of and organize number relations. These mathematically rich learning experiences should provide space for students' attention to being accurate, to being flexible in their thinking, and to developing efficiency in their reasoning and processes (Russell 2000).

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Fluency and CCSSM

In this article, I offer examples of learning tasks that build mathematical fluency to promote students' creative expression of mathematical ideas. These learning tasks align with the expectations of CCSSM, which mentions fluency twenty-five times and promotes fluency through problem solving and developing mathematical ideas. Teachers and districts can use **table 1** as a concise way of seeing the areas in which students ought to be fluent and when they are expected to obtain that fluency. To become mathematically fluent thinkers, students need multiple experiences of making sense of mathematical ideas and developing efficiencies based on their understandings.

Two ways that CCSSM intends for teachers to promote these experiences is through problem solving and the developmental challenge of modeling increasingly sophisticated mathematics scenarios. In solving word problems, students are expected to use physical objects, drawings, and equations to demonstrate their sense of the problem's solution. Problems that students are making sense of should involve mathematical ideas that they are expected to become fluent with. Ensuring that students reach the fluency mark for their grade level is not the only important goal. CCSSM mentions that students should use models to develop mathematical ideas and solve problems above the fluency level. Providing experiences for students to solve word problems of slightly higher difficulty is just as important as promoting their development of clear models and explanations for ideas that they will become fluent with during the next year. Fluency expectations, problem solving, and modeling new ideas work together in CCSSM to help K–grade 2 students scale up their fluency based on prior learning experiences (see **table 2**).

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CCSSM promotes the mathematical fluency that results when students continue to have increasingly challenging problem-solving experiences.

Learning experiences for fluency

In second grade, students will mentally be able to fluently add and subtract numbers within 20. Additionally, students will become fluent in

TABLE 1

This is not an exhaustive list of CCSSM expectations but rather a focused look at the requirements for student fluency.

K–Grade 2 CCSSM fluency expectations for addition and subtraction with whole numbers

Grade and standard		Addition	Subtraction
K	Fluency K.OA.5	Numbers within 1 to 5	
1	Fluency 1.OA.6 and 1.NBT.5	Numbers within 10 Mentally find 10 more or 10 less than a two-digit number.	
2	Fluency 2.NBT.5 and 2.OA.2	Use strategies of place value and properties to add and subtract numbers within 100. Mentally add and subtract numbers within 20.	

using place value and properties of operations to add and subtract numbers within 100. Students are also expected to engage in the eight Standards for Mathematical Practice (CCSSI 2010). To enable them to do so effectively, the classroom should have established norms and a classroom ecology that promotes safety in constructing and sharing viable arguments and critiquing others' thinking. I do not intend the order in the tasks that follow to describe a particular curriculum arrangement but rather to give an example of how Number of the Day, word problem solving, and modeling new mathematics problems align with CCSSM expectations to promote the development of fluency. These types of tasks and activities often coalesce to help students emerge mathematically fluent.

Number of the day

Mrs. Owens is a second-grade teacher who uses various mathematical tasks with her students to help them become fluent mathematical thinkers. She uses Number of the Day to explore mathematical relationships derived from

TABLE 2

At the K–2 grade level, fluency expectations, problem solving, and modeling new ideas work together in CCSSM to help students scale up their fluency based on previous learning experiences.

Aligning CCSSM fluency expectations with problem solving and developing ideas

Grade and standard		Addition	Subtraction
K	Fluency K.OA.5	Numbers within 1 to 5	
	Solving K.OA.2 and K.OA.3	Solve word problems by using objects or drawings and decompose numbers less than or equal to 10.	
	Modeling ideas K.NBT.1	Use objects or drawing to compose and decompose numbers from 11 to 19 into tens and ones.	
1	Fluency 1.OA.6 and 1.NBT.5	Numbers within 10	
		Mentally find 10 more or 10 less than a two-digit number.	
	Solving 1.OA.1	Solve word problems using various strategies for numbers within 20, including problems with three whole numbers, by using objects, drawings, and equations.	Solve word problems using various strategies for numbers within 20, by using objects, drawings, and equations.
	Modeling ideas 1.OA.2	Use models, properties, and place-value strategies to add a two-digit number to a one-digit number or add a two-digit number to a multiple of 10.	Use models, properties, and strategies to subtract one multiple of 10 from another multiple of 10 in the range of 10 to 90.
2	Fluency 2.NBT.5 and 2.OA.2	Use strategies of place value and properties to add and subtract numbers within 100.	
		Mentally add and subtract numbers within 20; know from memory all sums of two one-digit numbers.	
	Solving 2.OA.1	Solve word problems involving one or two steps for numbers within 100 by using drawings and equations.	
	Modeling ideas 2.NBT.7 and 2.NBT.8 and 2.OA.4	Use models, properties, and place value to add and subtract numbers within 1000.	
		Mentally add or subtract 10 or 100 to a number between 100 and 900.	

FIGURE 1

During this dialogue, the teacher encouraged students to think about properties and place value.

Teacher: We have Marcia's idea of 22 + 7 = 29. Can anyone besides Marcia share one other number sentence she might have written?

Denzell: She could write it 27 + 2 = 29. It's easier to add 2 on 27 than 7 on 22.

Marcia: I don't think that will work. That would take part of the number and move it instead of the whole thing. Like in 7 + 22 = 29?

Mary: Hmmm, I see Denzell's idea works, but like we were talking before, I don't know if it's a strategy.

Teacher: So, Mary is bringing up another thing we have been considering. Does an idea work in all situations? In your partner pairs, consider Denzell's idea and try to give reasons for why it will or will not always work. [*After a few minutes' work*] Let's come together and share ideas.

John: We found that Denzell is trading the two numbers, and that works for others [*He writes* 23 + 6 = 29 *and* 26 + 3 = 29 *on the whiteboard*.]

Mary: We also think it always works because he is not changing how much it is all together. It's just the ones.

Teacher: That's an interesting idea [*motioning for Mary's partner Shayna to continue discussing the idea*]. Shayna, why don't you tell us more about the ones. What does Mary mean?

Shayna: Well, we think that changing the numbers in the ones keeps the answer the same, but you can't do that with the tens.

Marcia: I guess that makes sense, because 22 + 7 is like 27 + 2, but 72 +2 would be way too big. It still seems weird switching just part of the number, though.

Kendrae: We switched the numbers, but we expanded first. Can we show? [*The teacher motions for Kendrae to write it on the board.*] So, we wrote Denzell's way [*He writes* 20 + 2 + 7 = 20 + 7 + 2]. Twenty-two is really 20 + 2. Then add 7 more. Or we can switch when we add the 2 and 7. It will always work because he is adding in a different order but adding the same numbers.

Marcia: Oh, yeah, I can see that now.

Teacher: I see a lot of heads nodding yes now. You all did a good job thinking about the place value in the strategy and using the commutative property to explain why it works.

[*Later in the year, the class revisited this discussion, deciding that you can switch the numbers in the ones place with each other and the numbers in the tens place with each other, but not across place values. The students further decided this was true for only addition and not for subtraction.*]

students' ideas. The activity generally takes from five to fifteen minutes. This day, she poses the number 29 and asks students to develop a number sentence in which 29 is the result. Owens then further challenges the students to use the same numbers in their sentence and make other number sentences. After students individually complete their idea, they share it with a partner and then have a whole-class discussion. Using students' ideas, Owens facilitates dialogue around making sense of one another's ideas and deepening mathematical understanding. During the conversation (see **fig. 1**), Owens encourages students to think about properties and place value as they address adjustments to Marcia's idea of 22 + 7 = 29. Although students typically choose to write other relationships, such as 7 + 22 = 29 or 29 – 7 = 22, what emerged this day from the whole-class discussion was different and significant.

When observing through the lens of fluency expectations in CCSSM for second grade, consider the following three important points in this classroom scenario:

1. Students build on their fluency from first grade by adding and subtracting numbers within 10, such as 2 + 7 or 7 + 2. This fluency allows them to attend to the more mathematically sophisticated work of explaining whether Denzell's idea always works.
2. Students use strategies of place value in their explanations as they note that Denzell can switch two numbers that are in the ones place but cannot switch a number in the ones place with a number in the tens place.
3. Students demonstrate understanding of the commutative property through their use of strategies and in their explanation of why Denzell's idea worked.

As the school year continues, Number of the Day will provide students with a steady flow of ways to consider the accuracy and efficiency of various strategies and become flexible in their thinking about adding and subtracting numbers within 100. As Owens works with her students, she will have ever more challenging expectations, so that as students reach the fluency requirement for second grade, their intellect will be challenged to think through addition and subtraction with larger numbers, exploring the viability of their strategies in new areas.

Word problem solving

Making sense of word problems and developing feasible solutions is highly valued in CCSSM. Through problem solving of this kind, students have even more opportunities to develop their expected grade-level fluency. In the middle of the year, as her class was still moving toward their second-grade fluency mark, Owens posed the following two-step problem on the board:

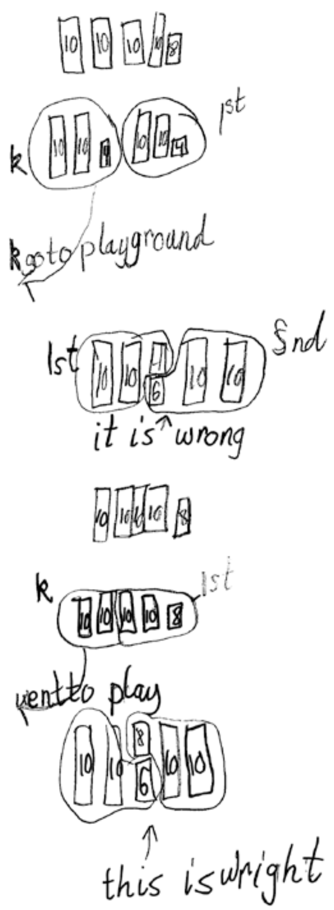
> There were 48 kindergartners and first graders all together in the cafeteria. The kindergartners left, and 26 second graders came in. Now there are 54 students in the cafeteria. How many kindergartners were in the cafeteria? How many first graders are in the cafeteria?

On receiving the problem, students set out to make sense of the scenario individually. After giving them sufficient time, Owens asks them to discuss their sense-making efforts with a partner. Several of the partner pairs decide to role-play a simpler scenario using blocks, rods, or counters. This action by the students emerges as a mathematical practice because Owens has encouraged them throughout the year to role-play problem scenarios and to experiment with smaller numbers as a way to persevere in problem solving.

Other students work to make complete concrete models, draw out the scenario making a sketch of their own cafeteria and tables, use guess-and-check strategies, and write equations they think may be useful. Next, each pair of students explains its solutions to a small group whose members determine whether the solution makes sense.

FIGURE 2

Organizing snapping blocks by units of 10 to represent the children in the problem, Elissa and Summer acted out the scenario repeatedly, practicing adding and subtracting numbers less than 100, the expected fluency mark.



"We first tried it out with blocks and then drew pictures. Since there were forty-eight, we made forty-eight like this: four tens and eight. Then took twenty-four kindergartners and twenty-four first graders. But the kindergartners left: 'Yay, we are going to the playground!'"

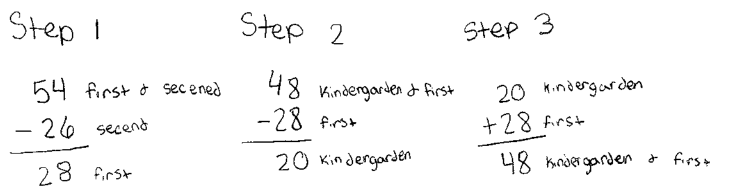
"Then there were twenty-four first graders and twenty-six second graders. But that was only fifty, not fifty-four. There are five tens. It was wrong. We tried many more times, like with twenty-six and twenty-two kindergartners, and we saw we needed to take out less kindergartners to get fifty-four."

"We found that twenty kindergartners went to the playground, yeah, were going to the playground. Then when the second grade came in, they added twenty-six to the twenty-eight. Six and eight make fourteen, and forty plus fourteen is fifty-four. That's how we do it."

By tackling the Number of the Day activity throughout the school year, second-grade students experienced a steady flow of ways to consider the accuracy and efficiency of various strategies, resulting in flexible mathematical thinking.

FIGURE 3

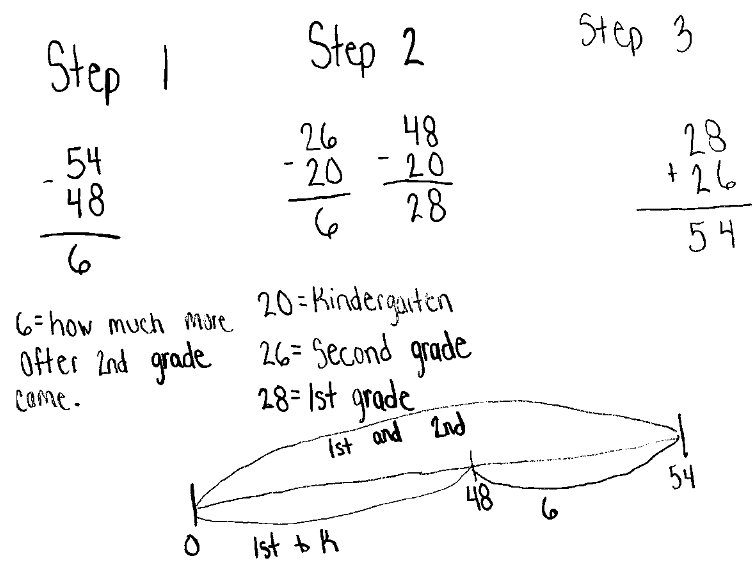
Kalley and Diana's working-backward strategy shows how they continue to use their first-grade fluencies of mentally adding or subtracting 10 from a two-digit number to build toward adding and subtracting fluently for numbers within 100.



"We did it by going backwards. There were fifty-four at the end, and twenty-six were second graders. So, taking twenty-six away, we said, fifty-four, forty-four, thirty-four, and six more was twenty-eight first graders in step one. Then at the beginning, there were forty-eight, and twenty-eight were first graders. So, forty-eight, thirty-eight, twenty-eight, and eight more was twenty kindergartners in step two. So, that's how we got the answer in step three."

FIGURE 4

Albert drew an open number line to successfully explain his and Bianca's strategy to students in their small group. As a result, they included the open number line in their whole-class explanation.



"Since the kindergartners left and the second grade came in [*pausing*], the total went up by six, so there are six more second graders. So, there must be twenty kindergartners; see? Twenty-six minus twenty is six. If there were forty-eight at the start, twenty were kindergartners, then forty-eight minus twenty is twenty-eight first graders. I don't know why we did step three. I don't think we need it."

In this particular class session, students agreed on three viable solution strategies (see **figs. 2**, **3**, and **4**). Their explanations are the results of students' perseverance and toil to find a solution. Elissa and Summer's work (see **fig. 2**) shows their use of snap cubes to represent the number of children in the problem. The girls organized the cubes by units of ten, acting out the scenario many times to figure out what needed to happen. Within their many attempts, they practiced a great deal of adding and subtracting numbers less than 100, the expected fluency mark. Owens encouraged her students to act out problems and situations as they needed to make sense of problems. In so doing, these second graders became occupied with learning mathematics and naturally worked toward building their fluency.

Kalley and Diana's solution (see **fig. 3**) cleverly attends to a strategy they had used a week before, working backward. Before presenting this problem, Owens highlighted the girls' working-backward strategy by posting their work with other solution types on the side wall of the classroom. The students often reference and use these strategy types when considering new problems. Kalley and Diana's explanation shows how they continue to use their first-grade fluencies of mentally adding or subtracting 10 from a two-digit number to build toward adding and subtracting fluently for numbers within 100.

Albert and Bianca (see **fig. 4**) show a great understanding of subtraction as the distance between two numbers on the number line. During Albert and Bianca's explanation to their small group, much confusion occurred around the number 6 and what it means. Albert decided to draw an open number line to help explain their solution. Students in the pair's small group saw the meaning of the 6 from the open number line. As a result, they included it in their whole-class explanation.

By making sense of and developing solutions to word problems, these students had the opportunity to develop flexible thinking and decide which kinds of thinking are accurate. Their work to solve the problem not only involved multiple additions and subtractions but also tasked students with creatively procuring a model and a strategy. The classroom culture that Owens established opens up opportunities for her students to develop fluency in creative mathematical expression.

Modeling new mathematical ideas

A great deal of importance should be placed on the activity of asking students to make sense of and model new mathematical ideas. Through the activity of modeling, students can come to see both why and how concepts connect, and by seeing these connections, they can invent abstract ways of handling the addition and subtraction of whole numbers. The following scenario from Owens's class shows the importance that modeling experiences can have in giving students opportunities to further their mathematical thinking with larger numbers.

Students in Owens's class had been working with base-ten blocks to represent adding two-digit numbers within 100. On this day, however, she asked the class to find a way to add 27 + 35 without using the blocks. In the challenge of moving away from a concrete representation, some students drew pictures, and others began to use the place-value meanings of the numerals. Owens noticed that Arnold used place-value ideas to find an answer, so she asked him to explain the idea to the class (see **fig. 5**). Arnold's explanation and representation show an understanding of place value.

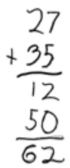
Arnold stated, "The 2 is really like two of the tens blocks, and the 3 is really like three of the tens blocks, so that's five tens blocks."

Arnold's experience with using blocks for modeling has helped him make sense of place-value meanings, but more important, Arnold offered for consideration his own creative mathematical expression using these understandings. His explanations directed other students' thinking toward the meaning of the numerals. Because students had been modeling these ideas in class, they had many experiences through which to connect the meanings of number representation and addition.

During the next couple of weeks, Owens noticed her students moving toward the efficiency of representing solutions to these problems by using the notion of place value. Because of this, she discerned that it was time to challenge her students with problems involving 2 two-digit numbers that sum to be more than 100. She gave students 46 + 58 as a first nudge

FIGURE 5

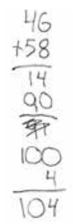
Noticing that Arnold's representation showed an understanding of place value, the teacher asked him to present his explanation to the class, which directed other students' thinking toward the meaning of the numerals.



"Seven plus five is twelve, so I wrote it down there. But this two is really twenty, right? And three is really thirty, so twenty plus thirty is fifty. Then I wrote it down, too. The one [*pointing to the twelve*] means ten, and five [*pointing to the fifty*] means fifty, so ten and fifty is sixty plus the two more."

FIGURE 6

Unsuccessfully trying to solve a two-digit problem on paper, Arnold reverted to modeling it with blocks and found his place-value error.



Arnold: I did this the same, but I messed up.

Owens: OK, explain what you did.

Arnold: I added 6 and 8, 14. Then 40 and 50, 90. At first, I saw 90 and 10 and wrote 91, but I knew that wasn't right, so I scratched it out and wrote 100, then added 4 more, 104.

into thinking beyond 100. Owens told students that they could use whatever tools they needed to help them come to a solution. She watched Arnold and the other students as they thought through the problem. At first, Arnold attempted to find the answer without the blocks. He started by trying to use place value and representations as he had before, but he got stuck and decided to grab some base-ten blocks. He modeled the new mathematics problem with the blocks, then quickly adapted his written representation to deal with the new value (see **fig. 6**).

Owens allowed Arnold time to think and the opportunity to select and use appropriate tools. Arnold's confession that he had first thought the sum was 91 shows how fragile student thinking is when students encounter problems at the abstract level that are more difficult to solve. Through the use of modeling, Arnold self-corrected his place-value error and adapted his previous ideas to meet the demand of the larger numbers. Later in the school year, these students reflected on these modeling experiences to help them adapt to solving problems involving 2 three-digit numbers by using models of blocks and drawings as well as properties and place value. Through these learning experiences, students came to sensibly choose, as Arnold did, the more efficient strategy of using place value to add and subtract large numbers. They were able to move from modeling with blocks to modeling with mathematical ideas. As students make their own choices about mathematical efficiency based on their own understandings, they develop a powerful fluency for quantitative thinking.

Fluency

Students become mathematically fluent thinkers when they have many occasions to make sense of problems and apply their understandings toward increasingly sophisticated problems. CCSSM expectations can guide teachers' instructional endeavors for the purpose of helping students become fluent. Such tasks as Number of the Day, word problems, and modeling new mathematical ideas align with CCSSM expectations and can be scaled to continually challenge students' thinking and create the space to develop accuracy, flexibility, and efficiency. Focused and coherent instruction using these ideas moves students beyond the mere ability to quickly recall facts into demonstrating fluency as a creative expression of mathematical ideas.

The learning scenarios described above focus on fluency development in second-grade students. For teachers who would like to see tasks and activities for kindergarten and first-grade students, I recommend the article "Fluency with Basic Addition" (Kling 2011), which gives ideas for developing early number sense, and "The Road to Fluency and the License to Think" (Buchholz 2004), which presents ideas on developing student strategies and mental thinking.

The qualities of people who fluently speak a foreign language are made evident as they actively engage with the language. Using both the structure and meanings within the language, they can create new forms of expression and new relationships. Imagine what it would mean to have a generation of students with similar quantitative ways of thinking and being. Fluency, as the ability to creatively express ideas, pertains to not only language but also mathematics. This meaning of fluency offers visions of students using both the structure of mathematics and the meanings of its representations. But more important, students would emerge from our classrooms with the quality of creative expression—that is, the ability to construct new associations and meanings with mathematics as they endeavor to solve the many problems and tasks encountered in our world.

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