Understanding and Teaching Fractions

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Goals for the presentation today on fractions

An opportunity to think together about:

- The Common Core definition of fraction and fractions on number lines;
- The reasoning underlying equivalent fractions;
- The transition from whole number to fraction multiplication;
- Connecting fractions with division;
- Attending closely to the wording of problems.

The Importance of a Focus on Fractions (Rational Numbers)

NCTM Focal Points

National Mathematics Advisory Panel

IES Practice Guide: Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

IES Practice Guide: Developing Effective Fractions Instruction for Kindergarten Through 8th Grade

Common Core State Standards for Mathematics

Common Core State Standards for Mathematics

Two domains for fractions and rational numbers:

- *Number and Operations—Fractions*, Grades 3 – 5
- *The Number System*, Grades 6 – 8.

3.NF *Number and Operations—Fractions*, Grade 3

3.NF Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

Definition of fraction

Why does the CC define fractions $\frac{A}{B}$ as A parts, each of size $\frac{1}{B}$?
Why are fractions *not* defined as "A out of B"?

The latter definition does not extend to improper fractions, such as $\frac{5}{4}$.
What would "5 out of 4" mean?

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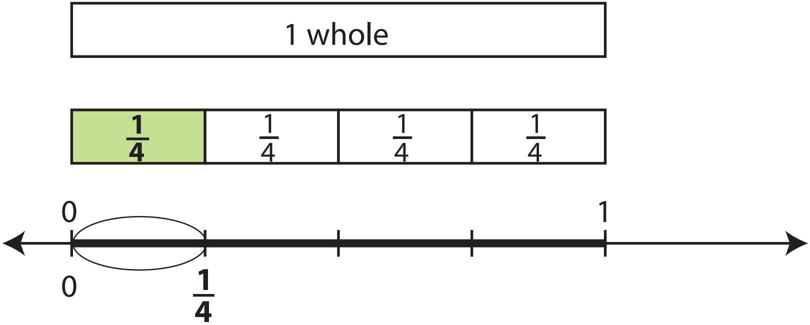
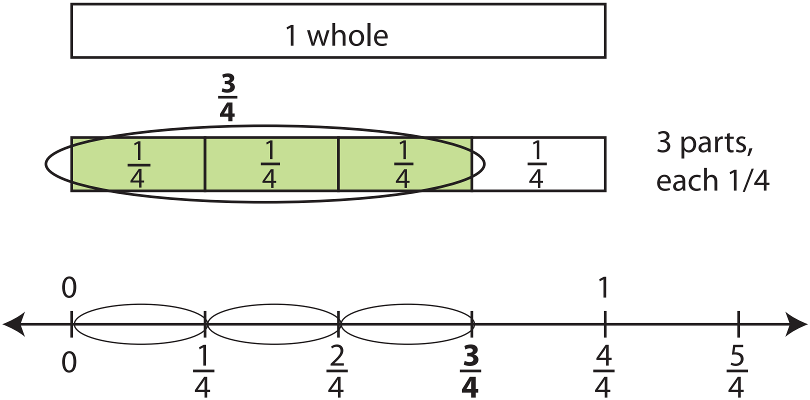
Definition of fraction

First define fractions with numerator 1 (unit fractions)

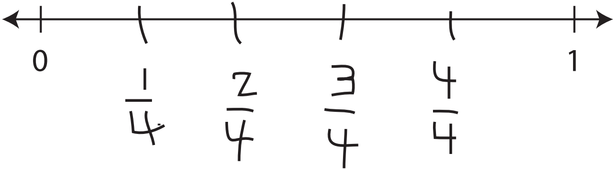


Definition of fraction



Watch out for errors with fractions on number lines

A common misconception:
students count tick marks instead of attending to length.



The student put 4 tick marks inside the interval instead of dividing the interval into 4 equal parts.

Definition of fraction

Your turn: Explain the definition of fraction to yourself or to a neighbor.

Show and describe: $\frac{1}{5}$ then $\frac{3}{5}$ then $\frac{6}{5}$ or

Show and describe: $\frac{1}{6}$ then $\frac{5}{6}$ then $\frac{7}{6}$

CC Fraction Definition:
The fraction $\frac{1}{B}$ is the quantity formed by 1 part when a whole is partitioned into B equal parts.
The fraction $\frac{A}{B}$ is the quantity formed by A parts of size $\frac{1}{B}$.

Visual representations to pave the way for the abstract version

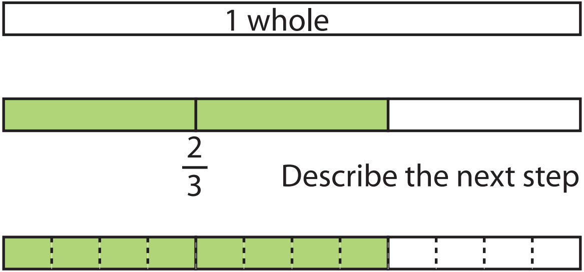
RtI Math, Recommendation 5 is on visual representations

“Use visual representations such as number paths, number lines, arrays, strip diagrams, other simple drawings or pictorial representations to scaffold learning and pave the way for understanding the abstract version of the representation.”

Equivalent fractions

4.NF.1
"Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions."

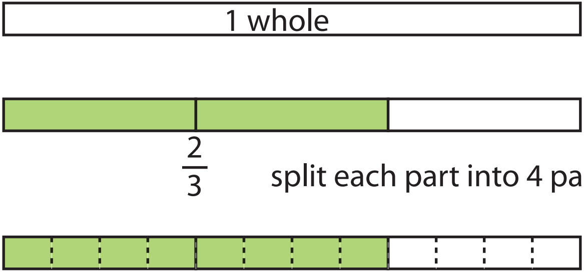
Equivalent fractions



Describe the next step

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Explain the 2×4
Explain the 3×4
Why $\times$, aren't we dividing?
Explain both $=$ signs



$$\frac{2}{3} = \frac{2\times 4}{3\times 4} = \frac{8}{12}$$

4 times as many shaded parts
4 times as many parts in all
more parts, but they are smaller
the *same amount* is shaded

Equivalent fractions in Grade 5

5.NF.5b
"...[relate] the principle of fraction equivalence a/b = (n$\times$a)/(n$\times$b) to the effect of multiplying a/b by 1."

For example:

$$\frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{4}{4} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

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Simple word problems give meaning to operations

RtI Math, Recommendation 4 is on solving word problems based on common underlying structures.

"Simple word problems give meaning to mathematical operations such as subtraction or multiplication. When students are taught the underlying structure of a word problem, they not only have greater success in problem solving but can also gain insight into the deeper mathematical ideas in word problems."

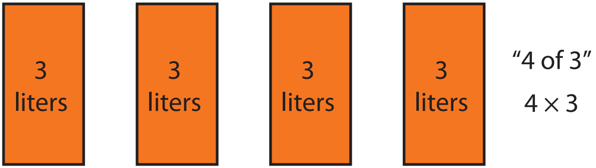
Fraction multiplication

5.NF Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3.OA.1:
"Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7."

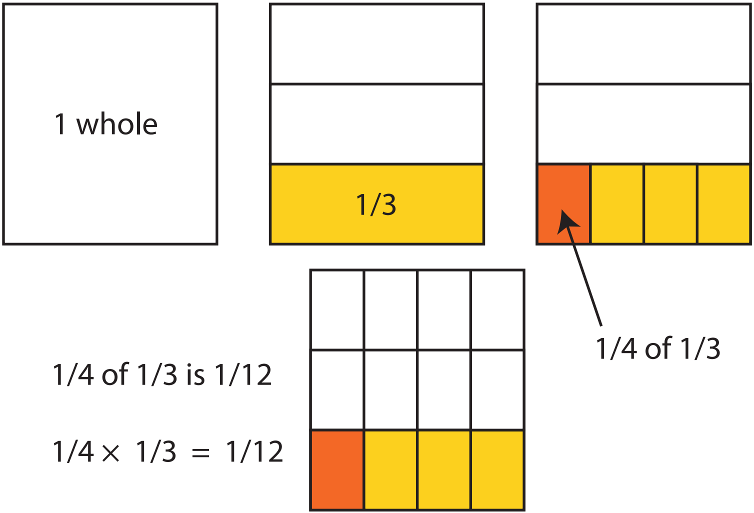
Extending multiplication to fractions

$A \times B$ is the amount in A groups of B each, "A of B."



Amount in 4 bottles, if a bottle is 3 liters	"4 of 3"	4×3 liters
Amount in 4 bottles, if a bottle is $\frac{1}{3}$ liter	"4 of $\frac{1}{3}$"	$4 \times \frac{1}{3}$ liters
Amount in $\frac{1}{4}$ bottle, if a bottle is $\frac{1}{3}$ liter	"$\frac{1}{4}$ of $\frac{1}{3}$"	$\frac{1}{4} \times \frac{1}{3}$ liters

Fraction multiplication



Connecting division and fractions

5.NF.3
"Interpret a fraction as division of the numerator by the denominator (a/b = a÷b)."

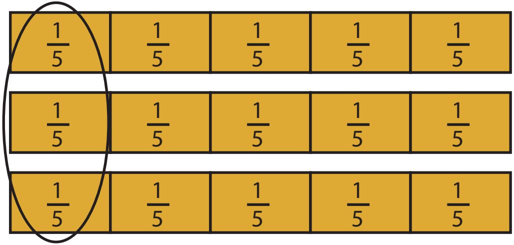
For example:

$$3 \div 5 = \frac{3}{5}$$

How can we see this relationship?

Connecting division and fractions

3 subs divided equally among 5 people



$$3 \div 5 = \frac{3}{5}$$

$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

1 person's share is $3 \div 5$

1 person's share is 3/5 of a sub

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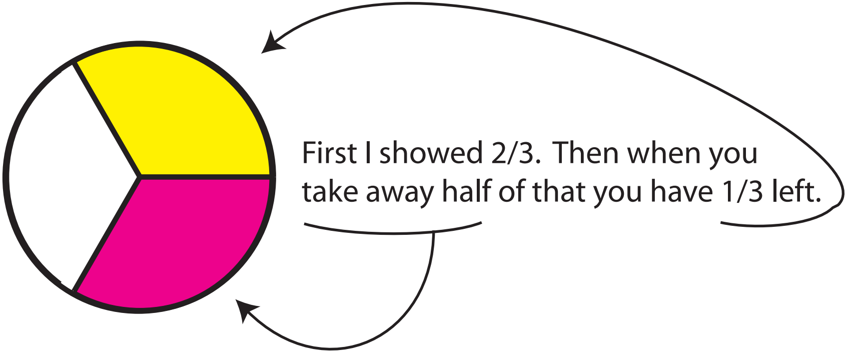
Is this a word problem for $\frac{2}{3} - \frac{1}{2}$?

There was $\frac{2}{3}$ of a pizza left over. Ben ate $\frac{1}{2}$ of the pizza that was left. Then how much pizza was left?



Attending closely to the wording of problems

Compare the wording, meaning, and way of solving these word problems:

1. Anna had $\frac{1}{2}$ cup of juice in her glass. She drank $\frac{1}{3}$ of it. How much juice is left?
2. Anna had $\frac{1}{2}$ cup of juice in her glass. She drank $\frac{1}{3}$ of a cup of juice. How much juice is left?
3. Anna had $\frac{1}{3}$ of a cup of juice in her glass. After she got some more juice, she had $\frac{1}{2}$ of a cup. How much more juice did she get?

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Thank you!

Questions? Comments?

[Problem 1 sounds like it's solved by $\frac{1}{2} - \frac{1}{3}$ but it isn't. Anna drank $\frac{1}{3}$ of $\frac{1}{2}$ of a cup, which is $\frac{1}{6}$ of a cup. She has $\frac{1}{2} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ of a cup left.
Problem 2 is solved by $\frac{1}{2} - \frac{1}{3}$. She has $\frac{1}{6}$ of a cup left.
Problem 3 sounds like it might involve addition, but is a $\frac{1}{3} + J = \frac{1}{2}$ problem, which is solved by $\frac{1}{2} - \frac{1}{3}$. Notice that the action in this problem reverses the action in the previous problem, so must have the same solution.]