



Appendix A:

Designing High School Traditional Mathematical Courses Based on the 2017 Kansas Mathematics Standards

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The 2017 Kansas Mathematics Standards are organized by grade level in Grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be a successful high school graduate in Kansas, and to be prepared to study more advanced mathematics. As schools consider how to implement the high school standards, an important consideration is how the standards might be organized into courses that provide a strong foundation for post-secondary success. To address this need, the Kansas Department of Education in partnership with members of the 2017 Mathematics Standards Committee have developed a Traditional Course Pathway in Mathematics based on the 2017 Kansas Mathematics Standards.

In considering this document, there are four things important to note:

1. The courses are models, not mandates. They illustrate possible approaches to organizing the content of the 2017 Kansas Mathematics Standards into coherent and rigorous courses that lead to post-secondary success. Districts and schools are not expected to adopt these courses as is; rather, they are encouraged to use this document as a starting point for developing their own.
2. All standards are found in the traditional pathway. The (+) standards are included to increase coherence but are not necessarily expected to be addressed on high stakes assessments. They should be limited to honors versions of the class or as extensions for those students who are ready for the content.
3. The course descriptions delineate the mathematics standards to be covered in a course; they are not prescriptions for curriculum or pedagogy. Additional work will be needed to create coherent instructional programs that help students achieve these standards.
4. While courses are given names for organizational purposes, districts and schools are encouraged to carefully consider the content in each course and use names that they feel are most appropriate.

Modeling

While the focus of this document is on organizing the Standards for Mathematical Content into a traditional pathway to graduation, the content standards must also be connected to the Standards for Mathematical Practice to ensure that the skills needed for later success are developed. In particular, Modeling (defined by a ★ in the 2017 Kansas Mathematics Standards) is defined as both a conceptual category for high school mathematics and a mathematical practice and is an important avenue for motivating students to study mathematics, for building their understanding of mathematics, and for preparing them for future success. Development of the pathways into instructional programs will require careful attention to modeling and the mathematical practices. Assessments based on these pathways should reflect both the content and mathematical practices standards.

Traditional Pathway: Algebra I

Introduction

The fundamental purpose of the Algebra I course is to formalize and extend the mathematics that students learned in the middle grades. For the high school Algebra I course, instructional time should focus on four critical areas:

1. In Algebra I, students learn to solve linear equations in one variable and apply graphical and algebraic methods to analyze and solve systems of linear equations in two variables. They analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.
2. In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and absolute value functions, including patterns; they interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students learn and apply properties of exponents to generate equivalent numerical and algebraic expressions. Students explore systems of linear and/or quadratic equations and linear inequalities, and they find and interpret their solutions.
3. Students apply the laws of exponents to integer exponents and recognize that squaring and cubing a number are inverse operations of taking the square root and cube root of a number; they strengthen their ability to see structure in and create quadratic expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students become facile with algebraic manipulation, including rearranging and collecting terms and factoring. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions such as absolute value.
4. Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze best fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Traditional Pathway: Geometry

Introduction

The fundamental purpose of the Geometry course is to formalize and extend students' geometric experiences from the middle grades. In this high school Geometry course, students explore more complex geometric situations and deepen their explanations of geometric relationships by presenting and hearing formal mathematical arguments. Important differences exist between this course and the historical approach taken in geometry classes. For example, transformations are emphasized in this course. Close attention should be paid to the introductory content for the Geometry conceptual category.

For the high school Geometry course, instructional time should focus on five critical areas:

1. Students develop experience with drawing triangles based on given measurements and performing rigid motions including translations, reflections, and rotations. They use these to develop notions about what it means for two objects to be congruent. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems— using a variety of formats including deductive and inductive reasoning and proof by contradiction—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.
2. Students apply their earlier experience with proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. They construct arguments about triangles using theorems. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity, and AA.
3. Students construct arguments about basic theorems related to circles, with particular attention to perpendicularity, in order to see symmetry in circles and as an application of triangle congruence criteria. They identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle as an application of similarity. They construct arguments using properties of polygons inscribed and circumscribed about circles.
4. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane. They use coordinates to prove simple geometric theorems algebraically, including the use of slope, distance, and midpoint formulas. Furthermore, they prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use the rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals, and slopes of parallel and perpendicular lines, which relates back to work done in the Algebra I course.
5. Apply geometric concepts in modeling situations. Apply concepts of density and displacement based on area and volume in modeling situations

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Traditional Pathway: Algebra II

Introduction

Building on their work with linear, quadratic, and absolute value functions, students extend their repertoire of functions to include exponential, logarithmic, polynomial, rational, and radical functions in the Algebra II course. Students work closely with the expressions that define the functions, are facile with algebraic manipulations of expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers.

For the high school Algebra II course, instructional time should focus on four critical areas:

1. A central theme of this Algebra II course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers.
2. Building on their previous work with functions they compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They graph square root, cube root, exponential and logarithmic functions, emphasizing the inverse relationship with exponentials and showing intercepts and end behavior. Additionally, they graph polynomial functions and identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.
3. Students synthesize and generalize what they have learned about a variety of function families. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function.
4. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this Algebra II course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Appendix III: High School Conceptual Category Tables

The mathematical content standards were designed for students to attain mathematical skills and concepts in a progression over time and across grade spans. The progressions were informed by research and by the logic of the mathematics. Conceptual categories are groups of inter-related standards. The tables describe how these conceptual category content standards are distributed across the high school courses. The conceptual category of Modeling is woven within the other five conceptual categories as it is best interpreted not as a collection of isolated topics but rather in relation to other standards.

Distribution of Content Standards by five Conceptual Categories:

1. Number and Quantity (N)
2. Algebra (A)
3. Functions (F)
4. Geometry (G)
5. Statistics and Probability (S)

Each Conceptual Category table shows the distribution of standards across the four High School Courses:

- Algebra I (AI)
- Geometry (GEO)
- Algebra II (AII)
- 4th Year Math (4th)

Focus Priority

Not all standards should have equal priority. At every level in mathematics there are intricate, challenging, and necessary concepts that serve as prerequisites for the next level's intricate, challenging, and necessary concepts. In order to help every student achieve success after high school both in the workforce and college we need to give them enough time to succeed in these major areas. The 2017 Kansas Math Standards were built from the mathematics learning progressions, so priorities were chosen with an eye to the arc of big ideas in the Standards. A plan of instructional focus that respects the learning progressions in the Standards will strike a balance between the path to gaining math understanding and the endpoint of having acquired it.

By using the Grade Level Focus (GLF) documents to inform your instructional practice there could be an approximate 70-20-10 breakdown of time across the three levels, Major, Supporting, and Additional. These numbers are not hard numbers and will fluctuate given the material contained in the various clusters at each grade level. There are times that the content within the Major clusters for a grade level call for more than 70% of the instructional time. Likewise, there are times where there is very little material in the Supporting clusters thus calling for less than 20% of the instructional time. With this in mind the rough 70-20-10 picture, 70 as a lower boundary while 10 is an upper bound.

It is important to note that while the three levels are mutually exclusive in the sense that each cluster belongs to one and only one level, in classroom implementation, the levels can work together to support the priorities of the grade. For example, teachers can view the Additional and Supporting levels in relation to the Major level by pulling the lower-priority material into a lesson that is centrally about more important ideas or topics; e.g., using geometric constructions to experiment with transformations.

Clusters comprised entirely of standards classified as "all" standards are included in this document. These standards should be taught throughout every high school math course, and often represent over-arching themes or key features of the mathematical concept. These standards should be taught in conjunction with the appropriate grade level standards.

Depth Opportunities (DO)

The DO's provide qualitative information about how to allocate time and effort sensibly within the Major Clusters and clusters essential to coherence from course to course, which is broad by itself. More specifically, they supplement the cluster-level priorities by highlighting beginnings, endings, or critical moments in learning progressions. They also give the prioritization a way to handle the highly uneven grain size of the content standards; many of the DO's are by no means small.

Like the cluster-level priorities, the DO's are meant to have relevance for curriculum, instruction, assessment, and professional development, and for the public and private sectors. For example, a curriculum designer producing a textbook chapter—or a teacher planning a unit of instruction—might opt to — “switch gears” for these areas, entering a more intense mode of engagement marked by tight focus, rigorous classroom reasoning and discussion, extended classroom time devoted to practice and reflection, and high expectations for mastery. Those reviewing or purchasing textbooks might pay particular attention to the quality of treatment in these areas. **One word of caution regarding the DO standards:** The idea that the identified DO standards might somehow be sufficient by themselves for instruction or assessment purposes is false. They serve the purpose of helping teachers increase the focus and rigor of their classrooms in a manner which aligns to the mathematical learning progressions.

Table Key:

- Major Focus Level (~70%): 
- Additional Focus Level (~10%) 
- Supporting Focus Level (~20%) 
- “All” Standards Focus Level **(all)**
- Included in Course **X**
- Modeling Standards **★**
- Depth Opportunities **DO**
- Boxes that are **yellow** are those that are considered extension topics for that course. They should be limited to honors versions of the class or as extensions for those students who are ready for the content.

Number and Quantity (N)

Cluster	Standard	A I	GEO	A II	4 th
The Real Number System (N.RN)					
A. Use properties of rational and irrational numbers. ● (9/10) ◆ (11)	N.RN.1	●			
	N.RN.2			◆	X
	N.RN.3			◆	X
Quantities (N.Q)					
A. Reason quantitatively and use units to solve problems. (all)	N.Q.1 (★) (all)	(all) DO			
	N.Q.2 (★) (all)	(all)	(all)	(all)	(all)
	N.Q.3 (★) (all)	(all)	(all)	(all)	(all)
A. The Complex Number System (N.CN)					
A. Perform arithmetic operations with complex numbers. ◆ (11)	N.CN.1			◆	
	N.CN.2			◆	
	N.CN.3			◆	
	N.CN.4 (+)				X
B. Represent complex numbers and their operations on the complex plane.	N.CN.5 (+)				X
	N.CN.6 (+)				X
	N.CN.7 (+)				X
C. Use complex numbers in polynomial identities and equations. ● (11)	N.CN.8			●	
	N.CN.9 (+)				X
	N.CN.10 (+)				X
Vector and Matrix Quantities (N.VM)					
A. Represent and model with vector quantities.	N.VM.1 (+)				X
	N.VM.2 (+)				X
	N.VM.3 (+)				X
	N.VM.4a (+)				X
	N.VM.4b (+)				X
	N.VM.4c (+)				X
	N.VM.5a (+)				X
	N.VM.5b (+)				X
B. Perform operations on matrices and use matrices in applications. ● (11)	N.VM.6			●	
	N.VM.7			●	
	N.VM.8			●	
	N.VM.9 (+)				X
	N.VM.10 (+)				X
	N.VM.11 (+)				X
	N.VM.12 (+)				X

Algebra (A)

Cluster	Standard	A I	GEO	A II	4 th
Seeing Structure in Expressions (A.SSE)					
A. Interpret the structure of expressions. (all)	A.SSE.1a (★) (all)	(all)	(all)	(all)	(all)
	A.SSE.1b (★) (all)	(all)	(all)	(all)	(all)
	A.SSE.2 (★) (all)	(all) DO	(all) DO	(all) DO	(all) DO
	A.SSE.3a (★)	▶ DO			
	A.SSE.3b (★)			▶ DO	
	A.SSE.3c (★)			▶ DO	
Arithmetic with Polynomials and Rational Expressions (A.APR)					
A. Perform arithmetic operations on polynomials. ◆ (9/10) ▶ (11)	A.APR.1	◆			× DO
	A.APR.2			▶	×
	A.APR.3			▶	×
B. Use polynomial identities to solve problems. ● (9/10) ● (11)	A.APR.4	●		●	
	A.APR.5 (+)				×
C. Rewrite rational expressions.	A.APR.6 (+)				×
	A.APR.7 (+)				×
Creating Equations (A.CED)					
A. Create equations that describe numbers or relationships. (all)	A.CED.1 (★) (all)	(all)	(all)	(all)	(all)
	A.CED.2 (★) (all)	(all) DO	(all) DO	(all) DO	(all) DO
	A.CED.3 (★) (all)	(all)	(all)	(all)	(all)
	A.CED.4 (★) (all)	(all)	(all)	(all)	(all)
Reasoning with Equations and Inequalities (A.REI)					
A. Understand solving equations as a process of reasoning and explain the reasoning. (all)	A.REI.1 (all)	(all)	(all)	(all)	(all)
B. Solve equations and inequalities in one variable. ▶ (9/10) ▶ (11)	A.REI.2 (all)	▶	▶	▶	(all)
	A.REI.3a	▶		▶	
	A.REI.3b (+)			×	×
	A.REI.4			▶	×
	A.REI.5a	▶ DO			
	A.REI.5b			▶ DO	× DO
	A.REI.5c			▶ DO	
	A.REI.5d (+)			▶ DO	× DO
	A.REI.6a	▶			
	A.REI.6b	▶			
	A.REI.6c	▶			
A.REI.7 (+)			×		
C. Represent and solve equations and inequalities graphically. ▶ (9/10) ▶ (11)	A.REI.8(all)	▶	▶	▶	(all)
	A.REI.9 (★)	▶		▶	▶
	A.REI.10	▶			

Functions (F)

Cluster	Standard	A I	GEO	A II	4 th
Interpreting Functions (F.IF)					
A. Understand the concept of a function and use function notation. ● (9/10) ◆ (11)	F.IF.1 (all)	●	●	◆	(all)
	F.IF.2 (all)	●	●	◆	(all)
	F.IF.3	●		◆	
B. Interpret functions that arise in applications in terms of the context. ◆ (9/10) ► (11)	F.IF.4 (★) (all)	◆	◆	► DO	(all) DO
	F.IF.5 (★) (all)	◆	◆	►	(all)
	F.IF.6 (★)	◆		►	×
	F.IF.7a (★)	►			
	F.IF.7b (★)			►	×
	F.IF.7c (★)			►	×
	F.IF.7d (+) (★)			►	×
	F.IF.7e			►	×
	F.IF.7f (+) (★)			►	×
	F.IF.7g (+) (★)			►	×
	F.IF.8a	► DO			
	F.IF.8b			► DO	×
	F.IF.8c			► DO	×
F.IF.9 (all)	► DO	► DO	► DO	(all) DO	
Building Functions (F.BF)					
A. Build a function that models a relationship between two quantities. ◆ (9/10) ► (11)	F.BF.1a	◆			
	F.BF.1b			►	×
	F.BF.1c			►	×
	F.BF.2 (+) (★)			►	×
B. Build new functions from existing functions. ● (9/10) ● (11)	F.BF.3	●		●	×
	F.BF.4a			●	×
	F.BF.4b			●	×
	F.BF.4c(+)			●	×
	F.BF.4d(+)			●	×
	F.BF.5			●	×
Linear, Quadratic, and Exponential Models (F.LQE)					
A. Construct and compare linear, quadratic, and exponential models and solve problems. ► (11)	F.LQE.1a (★)			► DO	
	F.LQE.1b (★)			►	
	F.LQE.1c (★)			►	
	F.LQE.2 (★)			►	
Trigonometric Functions F.TF					
A. Extend the domain of trigonometric functions using the unit circle.	F.TF.1(+)		×		×
	F.TF.2(+)		×		×
	F.TF.3(+)		×		×
	F.TF.4(+)		×		×
B. Model periodic phenomena with trigonometric functions.	F.TF.5 (+) (★)				×
	F.TF.6(+)				×
	F.TF.7(+)(★)				×
C. Prove and apply trigonometric identities.	F.TF.8(+)				×
	F.TF.9(+)				×

Geometry (G)

Cluster	Standard	A I	GEO	A II	4 th
Congruence (G.CO)					
A. Experiment with transformations in the plane. ▶ (9/10)	G.CO.1a		▶		
	G.CO.1b		▶		
	G.CO.1c		▶		
	G.CO.1d		▶		
	G.CO.2		▶		
B. Understand congruence in terms of rigid motions. ◆ (9/10)	G.CO.3		◆		
	G.CO.4		◆		
	G.CO.5 (+)		◆		
	G.CO.6 (+)		◆		
C. Construct arguments about geometric theorems using rigid transformations and/or logic. ▶ (9/10)	G.CO.7		▶		
	G.CO.8		▶		
	G.CO.9		▶		
	G.CO.10		▶		
D. Make geometric constructions. ● (9/10)	G.CO.11		●		
	G.CO.12 (+)		●		
Similarity, Right Triangles, and Trigonometry (G.SRT)					
A. Understand similarity in terms of similarity transformations. ● (9/10)	G.SRT.1a		●		
	G.SRT.1b		●		
	G.SRT.2		●		
	G.SRT.3		●		
	G.SRT.4		●		
B. Construct arguments about theorems involving similarity. ◆ (9/10)	G.SRT.5		◆		
	G.SRT.6		◆		
C. Define trigonometric ratios and solve problems involving right triangles. ◆ (9/10)	G.SRT.7		◆		
	G.SRT.8 (★)		◆		
	G.SRT.9		◆		
D. Apply trigonometry to general triangles.	G.SRT.10 (+)		×		×
	G.SRT.11 (+)		×		
	G.SRT.12 (+)		×		
Circles (G.C)					
A. Understand and apply theorems about circles. ● (9/10)	G.C.1		●		
	G.C.2		●		
	G.C.3		●		
	G.C.4 (+)		●		×
	G.C.5 (+)		●		×
B. Find arc lengths and areas of sectors of circles.	G.C.6 (+)		×		×
Expressing Geometric Properties with Equations (G.GPE)					
A. Translate between the geometric description and the equation for a conic section. ● (9/10)	G.GPE.1		●		
	G.GPE.2 (+)			●	
	G.GPE.3 (+)			●	×
	G.GPE.4 (+)			●	×
	G.GPE.5 (+)			●	×
B. Use coordinates to prove simple geometric theorems algebraically. ▶ (9/10)	G.GPE.6		▶ DO		
	G.GPE.7		▶		
	G.GPE.8 (★)		▶ DO		

Geometric Measurement and Dimension (G.GMD)					
A. Explain volume formulas and use them to solve problems.	G.GMD.1 (+)		✗		✗
	G.GMD.2 (+)		✗		✗
Modeling with Geometry (G.MG)					
A. Apply geometric concepts in modeling situations. ▶ (9/10)	G.MG.1 (★)		▶		
	G.MG.2 (★)		▶ DO		
	G.MG.3 (★)		▶		

Statistics and Probability (S)

Cluster	Standard	A I	GEO	A II	4 th
Interpreting Categorical and Quantitative Data (S.ID)					
A. Summarize, represent, and interpret data on a single count or measurement variable. ▶ (9/10)	S.ID.1	▶			
	S.ID.2	▶ DO			
	S.ID.3 (+)			×	×
B. Summarize, represent, and interpret data on two categorical and quantitative variables. ◆ (9/10)	S.ID.4	◆			
	S.ID.5a	◆			
	S.ID.5b	◆			
	S.ID.5c (+)				×
	S.ID.5d (+)				×
C. Interpret linear models. ▶ (9/10) ● (11)	S.ID.6	▶			
	S.ID.7			●	×
	S.ID.8			●	×
Making Inferences and Justifying Conclusions (S.IC)					
A. Understand and evaluate random processes underlying statistical experiments.	S.IC.1 (+)				×
	S.IC.2 (+)				×
B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	S.IC.3 (+)				×
	S.IC.4 (+)				×
	S.IC.5 (+)				×
	S.IC.6 (+)				×
Conditional Probability and the Rules of Probability (S.CP)					
A. Understand independence and conditional probability and use them to interpret data.	S.CP.1 (+)		×	×	×
	S.CP.2 (+)		×	×	×
	S.CP.3 (+)		×	×	×
	S.CP.4 (+)		×	×	×
	S.CP.5 (+)		×	×	×
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.	S.CP.6 (+)		×	×	×
	S.CP.7 (+)		×	×	×
	S.CP.8 (+)		×	×	×
	S.CP.9 (+)		×	×	×
Using Probability to Make Decisions (S.MD)					
A. Calculate expected values and use them to solve problems.	S.MD.1 (+) (★)				×
	S.MD.2 (+) (★)				×
	S.MD.3 (+) (★)				×
	S.MD.4 (+) (★)				×
B. Use probability to evaluate outcomes of decisions.	S.MD.5a (+) (★)				×
	S.MD.5b (+) (★)				×
	S.MD.6 (+) (★)		×	×	×
	S.MD.7 (+) (★)		×	×	×