Kansas College Career Ready Standards for Mathematics Flip Book for Grade 4

Updated Fall, 2014

This project used the work done by the Departments of Educations in Ohio, North Carolina, Georgia, engageNY, NCTM, and the Tools for the Common Core Standards.

About the Flip Books

The development of the "flip books" is in response to the adoption of the Common Core State Standards by the state of Kansas in 2010. Teachers who were beginning the transition to the new Kansas Standards—Kansas College and Career Ready Standards (KCCRS) needed a reliable starting place that contained information and examples related to the new standards.

This project attempts to pull together, in one document some of the most valuable resources that help develop the intent, the understanding and the implementation of the KCCRS. The intent of these documents is to provide a starting point for teachers and administrators to begin unraveling the standard and is by no means the only necessary or complete resource that supports implementation of KCCRS.

This project began in the summer 2012 with the work of Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books". The "flip books" are based on a model that Kansas had for earlier standards however, this edition is far more comprehensive than those in the past. The current editions incorporate the resources from: other state departments of education, documents such as the content progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. The current product was a compilation of work from the project developers in conjunction with many mathematics educators from around the state. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KATM website at www.katm.org and will continue to undergo changes periodically. When significant changes/additions are implemented the necessary modification will be posted and dated.

The initial development of the current update to the "flip books" was driven by the need expressed by teachers of mathematics in Kansas and with the financial support from Kansas Department of Education and encouragement from the Kansas Association of Teachers of Mathematics. These "flip books" have become an ongoing resource that will continue to evolve as more is learned about high quality instruction for the KCCRS for mathematics.

For questions or comments about the flipbooks please contact Melisa Hancock at melisa@ksu.edu.

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptually understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in "Additional and Sample clusters". Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with "+" within the standards. These standards fall outside of priority recommendations.

Recommendations for using cluster level priorities

Appropriate Use:
- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:
- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters' headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.

DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus**, **rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. (See Depth of Knowledge (DOK) Table 6, Appendix).

In this document, Depth Opportunities are highlighted pink in the Standards section. For example:

5.NBT.6 *Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.*

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Grade 4

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 4 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 4 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to their work with fractions and decimals. This involves two processes- decontexualizing and contextualizing. Grade 4 students decontextualize by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, "*if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed?* Students will decontextualize by writing the equation $\frac{3}{8} \times 5$ or repeatedly add $\frac{3}{8}$ five times. While students are working they will contextualize their work- knowing that the answer $\frac{15}{8}$ or $1\frac{7}{8}$ represents the total number of pounds of roast beef that will be needed. Further, Grade 4 students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 4 construct arguments using concrete representations, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. Students refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking through discussions and written responses.
4) Model with mathematics.	Mathematically proficient students in Grade 4 represent problem situations in various ways, including writing an equation to describe the problem. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Grade 4 students should evaluate their results in the context of the situation and reflect on whether the results make sense.

Practice	Explanation and Example
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 4 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6) Attend to precision.	Mathematically proficient students in Grade 4 develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7) Look for and make use of structure.	Mathematically proficient students in Grade 4 closely examine numbers to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 4 notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; "Does this make sense?"	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _____ related to _____? • What is the relationship between _____ and _____? • What does _____ mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? How can we be sure that _____? / How could you prove that._____? Will it still work if._____? • What were you considering when._____? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about._____? • How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. **Use appropriate tools strategically.** • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative? • Why was it helpful to use._____? • What can using a _____ show us, that ____may not? • In what situations might it be more informative or helpful to use._____?
6. **Attend to precision.** • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Is there a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties can you use to explain._____? • How could you test your solution to see if it answers the problem?
7. **Look for and make use of structure.** • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	• What observations do you make about._____? • What do you notice when._____? • What parts of the problem might you eliminate, simplify? • What patterns do you find in._____? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to._____? • In what ways does this problem connect to other mathematical concepts?
8. **Look for and express regularity in repeated reasoning.** • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Will the same strategy work in other situations? • Is this always true, sometimes true or never true? • How would we prove that._____? • What do you notice about._____? • What is happening in this situation? • What would happen if._____? • What Is there a mathematical rule for._____? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
   (OA.1, OA.2, OA.3, NBT.1, NBT .2, NBT.3, NBT.4, NBT.5, NBT. 6

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $\frac{15}{9} = \frac{5}{3}$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
   (NF.1,NF.2,NF3,NF.4,NF.5,NF.6,NF.7)

3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.
   (G.1, G.2, G.3)

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Grade 4 Content Standards Overview

Operations and Algebraic Thinking (OA)

- Use the four operations with whole numbers to solve problems.
  OA.1   OA.2   OA.3
- Gain familiarity with factors and multiples.
  OA.4
- Generate and analyze patterns.
  OA.5

Number and Operations in Base Ten (NBT)

- Generalize place value understanding.
  NBT.1   NBT.2   NBT.3
- Use place value understanding and properties of operations to perform multi-digit arithmetic.
  NBT.4   NBT.5   NBT.6

Number and Operations—Fractions (NF)

- Extend understanding of fraction equivalence and ordering.
  NF.1   NF.2
- Build fractions from unit fractions by applying and extending pervious understandings of operations on whole numbers.
  NF.3   NF.4   NF.5   NF.6   NF.7

Measurement and Data (MD)

- Solve problems involving measurement and conversions of measurements from larger units to smaller units.
  MD.1   MD.2   MD.3
- Represent and interpret data.
  MD.4
- Geometric measurement: understand concepts of angles and measure angles.
  MD.5   MD.6   MD.7

Geometry (GE)

- Draw and identify lines and angle, classify shapes by properties of their lines and angles.
  G.1   G.2   G.3

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Operations and Algebraic Thinking   (OA)

Cluster: *Uses the four operations with whole numbers to solve problems.*

Standard: Grade 4.OA.1

Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.4   Model with mathematics.
- ✓ MP.6   Attend to precision

Connections: (4.OA. 1-3):

This cluster is connected to:

- Fourth Grade Critical Area of Focus #1. **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.**
- Represent and solve problems involving multiplication and division (Grade 3 OA 3).
- Solve problems involving the four operations, and identify and explain patterns in arithmetic (Grade 3 OA 8).

Explanation and Examples:

A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be given many opportunities to write and identify equations and statements for multiplicative comparisons. It is essential that students are provided many opportunities to solve contextual problems.

Example:

Multiplicative Comparison
5 x 8 = 40.
Sally is five years old. Her mom is eight times older. How old is Sally's Mom?

5 x 5 = 25
Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

For More Information **See Learning Progressions.**

Instructional Strategies: (4.OA.1-3)

Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Major   Supporting   Additional   Depth Opportunities(DO)

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Appendix, Table 2. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2).

Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Resources/Tools:
Table 2 (Appendix, page 83)

For detailed information see Learning Progression Operations and Algebraic Thinking:
http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

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4.OA Comparing Growth, Variation 1
4.OA Comparing Growth, Variation 2
4.NBT.1 Threatened and Endangered
4.NBT Thousands and Millions of Fourth Graders

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Examples of multistep word problems can be accessed from the released questions on the NAEP (National Assessment of Educational Progress) Assessment.

For example, a constructed response question from the 2007 Grade 4 NAEP assessment reads, *"Five classes are going on a bus trip and each class has 21 students. If each bus holds only 40 students, how many buses are needed for the trip?"*

Common Misconceptions:
Students may have "overspecialized" their knowledge of multiplication or division facts and have restricted it to "fact tests" or one particular format. For example they may think of Multiplicative comparisons, unknown product or partition unknown (see Table 2 Appendix, page 83). For example students complete multiplication fact assessments satisfactorily but cannot apply knowledge to problem solving situations.

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Use the four operations with whole numbers to solve problems.*

Standard: Grade 4.OA.2

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (see Table 2 Appendix, page 83)

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.OA.1

Explanation and Examples:

This standard calls for students to translate comparative situations into equations with an unknown and solve.

Students need many opportunities to solve contextual problems. Refer to Table 2, Appendix, for more examples.

Examples:

Unknown Product: A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? $(3 \times 6 = p)$.

Group Size Unknown: A book costs $18. That is 3 times more than a DVD. How much does a DVD cost? $(18 \div p = 3 \ or \ 3 \times p = 18)$.

Number of Groups Unknown: A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? $(18 \div 6 = p \ or \ 6 \times p = 18)$.

When distinguishing multiplicative comparison from additive comparison, students should note that:

- *Additive comparisons* focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, "How many more?"
- *Multiplicative comparisons* focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is "How many times as much?" or "How many times as many?"

tudents need many opportunities to solve contextual problems. Table 2in the Appendix, includes the following multiplication problem:

"A blue hat costs $6. A red hat costs 3 times as much as the blue hat.
How much does the red hat cost?"

In solving this problem, the student should identify $6 as the quantity that is being multiplied by 3. The student should write the problem using a symbol to represent the unknown.
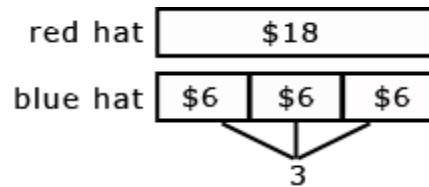
$$\$6 \times 3 = \square$$

red hat	$18		
blue hat	$6	$6	$6

3

Table 2 includes the following division problem:

A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?

In solving this problem, the student should identify $18 as the quantity being divided into shares of $6.

The student should write the problem using a symbol to represent the unknown.

$$\$18 \div \$6 = \square$$

blue hat	$6		
red hat	$6	$6	$6

$18
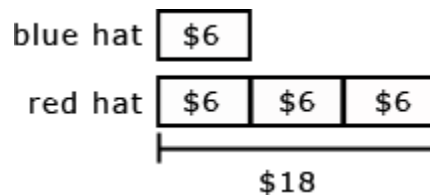
Instructional Strategies: See Grade 4.OA.1

Tools/Resources:
4.OA Comparing Money Raised

See also: National Assessment of Educational Progress.

Common Misconceptions: See Grade 4.OA.1

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Use the four operations with whole numbers to solve problems.*

Standard: Grade 4.OA.3

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4. OA.1

Explanation and Examples:

The focus in this standard is to have students use and discuss various strategies. It refers to **estimation** strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total? Some typical estimation strategies for this problem:

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Creighton brings in 3 packs with 6 bottles in each container. Susan wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	Student 2
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300 - 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:

Write different word problems involving $44 \div 6 = ?$ where the answers are best represented as:

- Problem A: 7
- Problem B: $7r2$
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\frac{2}{6}$

Possible solutions:

Problem A: 7.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill?

$$44 \div 6 = p; p = 7r2. \text{ Mary can fill 7 pouches completely.}$$

Problem B: $7r2$.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left?

$$44 \div 6 = p; p = 7r2; \text{ Mary can fill 7 pouches and have 2 left over.}$$

Problem C: 8.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils?

$$44 \div 6 = p; p = 7r2; \text{ Mary can needs 8 pouches to hold all of the pencils.}$$

Problem D: 7 or 8.

Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received?

$$44 \div 6 = p; p = 7r2;$$ *Some of her friends received 7 pencils. Two friends received 8 pencils.*

Problem E: $7\frac{2}{6}$

Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled?

$$44 \div 6 = p; p = 7\frac{2}{6}.$$

Example:

There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *($128 \div 30 = b$; $b = 4r8$; They will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over.

Example:

Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?

$$3 \times \$12 + \$15 = a$$

In division problems, the remainder is the whole number left over when as large a multiple of the divisor as possible has been subtracted.

Example:

Kim is making candy bags. There will be 5 pieces of candy in each bag. She had 53 pieces of candy. She ate 14 pieces of candy. How many candy bags can Kim make now?

(7 bags with 4 leftover)

Kim has 28 cookies. She wants to share them equally between herself and 3 friends. How many cookies will each person get?

(7 cookies each) 28 ÷ 4 = *a*

There are 29 students in one class and 28 students in another class going on a field trip. Each car can hold 5 students. How many cars are needed to get all the students to the field trip?

(12 cars, one possible explanation is 11 cars holding 5 students and the 12th holding the remaining 2 students)

$$29 + 28 = 11 \times 5 + 2$$

Major   Supporting   Additional   Depth Opportunities(DO)

Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

Students need many opportunities solving multistep story problems using all four operations and ALL *situations* found in Tables 1 and 2, Appendix.

An interactive whiteboard, document camera, drawings, words, numbers, and/or objects may be used to help solve story problems.

Tools/Resources:
4.OA, MD Karl's Garden
4.OA Carnival Tickets

Common Misconceptions:
Students apply a procedure that results in remainders that are expressed as r or R for ALL situations, even for those for which the result does not make sense. For example when a student is asked to solve the following problem, the student responds to the problem—there are 32 students in a class canoe trip. They plan to have 3 students in each canoe. How many canoes will they need so that everyone can participate? And the student answers of "$10r2$ canoes".

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Gain familiarity with factors and multiples.*

Standard: Grade 4.OA.4

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.7   Look for and make use of structure.

Connections:

This cluster is connected to:

- Fourth Grade Critical Area of Focus #1, Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.
- Understand properties of multiplication and the relationship between multiplication and division (Grade 3 OA 5 – 6).
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition (Grade 3 MD 7a).
- The concepts of prime, factor and multiple are important in the study of relationships found among the natural numbers. Compute fluently with multi-digit numbers and find common factors and multiples (Grade 6 NS 4).

Explanation and Examples:

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite**

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and the number itself. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Major   Supporting   Additional   Depth Opportunities(DO)

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors.

Students investigate whether numbers are prime or composite by
- building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
- finding factors of the number

Students should understand the process of finding factor pairs so they can do this for any number 1 – 100.

Example:

Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Multiples: 1, 2, 3, 4, 5…24

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

3, 6, 9, 12, 15, 18, 21, 24

4, 8, 12, 16, 20, 24

8, 16, 24

12, 24

24

To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following:
- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

Instructional Strategies:

Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counter-examples of factors.

Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12 . . . are composite.

Major   Supporting   Additional   Depth Opportunities(DO)

After working with the numbers 1 to 20, consider using a hundreds chart and have the students color code multiples of numbers. The color will help students see emerging patterns which they can discuss.

Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number.

Using area models will also enable students to analyze numbers and arrive at an understanding of whether a number is prime or composite. Have students construct rectangles with an area equal to a given number.

They should see an association between the number of rectangles and the given number for the area as to whether this number is a prime or composite number.

Definitions of prime and composite numbers should **not** be provided, but determined after many strategies have been used in finding all possible factors of a number.

Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets. For example, have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers.

Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. For example if you have 16 squares:
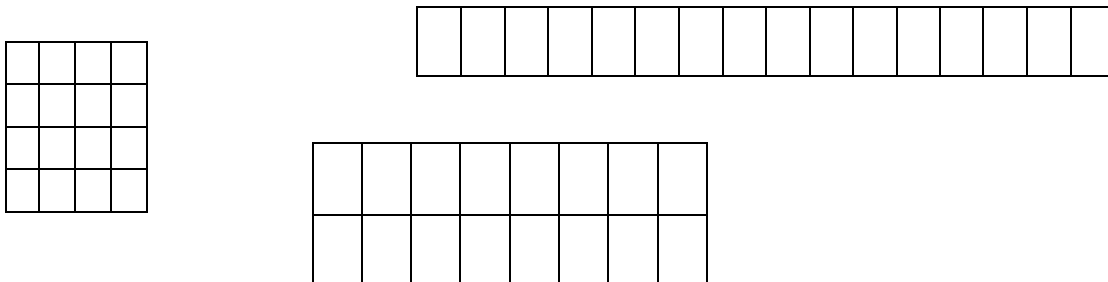
The idea that a product of any two whole numbers is a common multiple of those two numbers is a difficult concept to understand. For example, 5 x 8 is 40; the table below shows the multiples of each factor.

5	10	15	20	25	30	25	40	45
8	16	24	32	40	48	56	64	72

Ask students what they notice about the number 40 in each set of multiples; 40 is the 8th multiple of 5, and the 5th multiple of 8.

Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in Grade 6.

Writing multiplication expressions for numbers with several factors and for numbers with a few factors will help students in making conjectures about the numbers. Students need to look for commonalities among the numbers.

Major   Supporting   Additional   Depth Opportunities(DO)

Resources/Tools

- [4.OA Identifying Multiples](#)
- [4.OA Numbers in a Multiplication Table](#)
- [4.OA Multiples of 3, 6, and 7](#)
- [4.OA The Locker Game](#)

"The Factor Game ", NCTM.org <u>Illuminations</u>. This engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions of prime and composite numbers.

[National Center for Education Statistics: NAEP Questions Tool](#)

["The Product Game– Classifying Numbers ",NCTM.org *Illuminations*](#). Students construct Venn diagrams to show the relationships between the factors or products of two or more numbers in the game.

["Multiplication: It's in The Cards", (NCTM.org (Illuminations)](#). Patterns with Products.

[National Library of Virtual Manipulatives](#). (There may be an associated cost for this site)

Common Misconceptions:

When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself. Also, having students write multiples of a number by consecutive factors beginning with one can clear up this misconception.

Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.

Domain: Operations and Algebraic Thinking (OA)
Cluster: *Generate and analyze patterns.*

Standard: Grade 4.OA.5
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:
- This cluster goes beyond the Fourth Grade Critical Areas of Focus to address **Analyzing patterns**.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic. (3.OA.4.9).

Explanation and Examples:
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **pattern (number or shape), pattern rule.**

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Examples:

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23, 28…	Start with 3, add 5	The numbers alternately end with a 3 or 8
5, 10, 15, 20…	Start with 5, add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

Major   Supporting   Additional   Depth Opportunities(DO)

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:
Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers

$$(3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18)$$

In this standard, students **describe** features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5.

Day	Operation	Beans
0	3x0+4	4
1	3x1+4	7
2	3x2+4	10
3	3x3+4	13
4	3x4+4	16
5	3x5+4	19

Instructional Strategies:
In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make **generalizations**.

As students generate numeric patterns for rules, they should be able to "undo" the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, "Add 4" starting with the number 1, the pattern 1, 5, 9, 13, 17, … is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, "How is a number in the sequence related to the one that came before it?", and "If they started at the end of the pattern, will this relationship be the same?" Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Students should also determine if there are other relationships in the patterns. In the numeric pattern generated above, students should observe that the numbers are all odd numbers.

Provide patterns that involve shapes so that students can determine the rule for the pattern. For example,
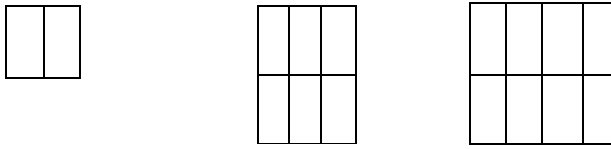
Students may state that the rule is to multiply the previous number of squares by 3.

Tools/Resources
4.OA Multiples of 3, 6, and 7
4.OA Double Plus One
4.OA Multiples of nine

"Snake Patterns –s-s-s", PBS Teachers. Students will use given rules to generate several stages of a pattern and will be able to predict the outcome for any stage.

"Patterns that Grow – Growing Patterns", NCTM.org Illuminations. Students use numbers to make growing patterns. They create, analyze, and describe growing patterns and then record them. They also analyze a special growing pattern called Pascal's triangle.

"Patterns that Grow – Exploring Other Number Patterns", NCTM.org Illuminations. Students analyze numeric patterns, including Fibonacci numbers. They also describe numeric patterns and then record them in table form.

"Patterns that Grow – Looking Back and Moving Forward ", NCTM.org Illuminations. In this final lesson of the unit, students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts.

Common Misconceptions:
Students think that results are random. There is no pattern. Another common misconception when students are working with repeating patterns is that they will often repeat what is given rather than looking at what "chunks" or part of the pattern is actually being repeated. Example: Given the pattern 6,9,12,6,9,12,6,9,… If the student is asked "*what is the next number in the pattern*", they may respond with "6" because they are returning to the beginning of the given pattern and repeat it from there. Students should be encouraged to look for the repeating "set".

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations Base Ten (NBT)

Cluster: *Generalize place value understanding for multi-digit numbers.*

Standard: Grade 4.NBT.1

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: (4.NBT.1-3)

This cluster is connected to:
- Fourth Grade Critical Area of Focus #1, Developing an understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends.
- A strong foundation in whole-number place value and rounding is critical for the expansion to decimal place value and decimal rounding.
- Understand place value (Grade 2 NBT 1 – 4).
- Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 3 NBT 1).

Explanation and Examples:

Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Mathematically proficient students communicate precisely by engagingĝ in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round**

This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

Example:

How is the 2 in the number 582 similar to and different from the 2 in the number 528?
Students should be familiar with and use place value as they work with numbers. Some activities that will help students develop understanding of this standard are:

- Investigate the product of 10 and any number, then justify why the number now has a 0 at the end. ($7 \times 10 = 70$ because 70 represents 7 tens and no ones, $10 \times 35 = 350$ because the 3 in 350 represents 3 hundreds, which is 10 times as much as 3 tens, and the 5 represents 5 tens, which is 10 times as much as 5 ones.) While students can easily see the pattern of adding a 0 at the end of a number when multiplying by 10, they need to be able to justify why this works.
- Investigate the pattern, 6, 60, 600, 6,000, 60,000, 600,000 by dividing each number by the previous number.

Major   Supporting   Additional   Depth Opportunities(DO)

Instructional Strategies: (4.NBT.1-3)

Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers.

Students need to have opportunities to compare numbers with the same number of digits, e.g., compare 453, 698 and 215; numbers that have the same number in the leading digit position, e.g., compare 45, 495 and 41,223; and numbers that have different numbers of digits and different leading digits, e.g., compare 312, 95, 5245 and 10,002.

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

In Grade 4, rounding should not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value.

What is new for Grade 4 is rounding to digits other than the leading digit, e.g., round 23,960 to the nearest hundred. This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1,000, not just zero.

Students should also begin to develop some rules for rounding, building off the basic strategy of; "Is 48 closer to 40 or 50?" Since 48 is only 2 away from 50 and 8 away from 40, 48 would round to 50. Number Lines are affective tools here. Then, students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit.

Resources/Tools

For detailed information see Learning Progression Number and Operations in Base Ten

4.NBT What's My Number?
4.NBT.1 Threatened and Endangered
4.NBT Thousands and Millions of Fourth Graders

Common Misconceptions: (4.NBT.1-3)

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students.

Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two).

There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method.

Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Generalize place value understanding for multi-digit whole numbers.*

Standard: Grade 4.NBT.2

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NBT.1

Explanation and Examples:

This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five.   However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.

Students should also be able to compare two multi-digit whole numbers using appropriate symbols.

The expanded form of 275 is 200 + 70 + 5. Students use place value to compare numbers. For example, in comparing 34,570 and 34,192, a student might say, both numbers have the same value of 10,000s and the same value of 1000s however, the value in the 100s place is different so that is where I would compare the two numbers.

Instructional Strategies: See Grade 4 NBT.1

Resources/Tools:

4.NBT Ordering 4-digit numbers

Common Misconceptions: See Grade 4 NBT.1

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations in Base Ten (NBT)

Standard: Grade 4.NBT.3

Use place value understanding to round multi-digit whole numbers to any place.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure

Connections: See Grade 4.NBT.1

Explanation and Examples:

This standard refers to place value understanding, which extends **beyond** an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	Student 2
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40+20=60. 300- 60 = 240, so we need about 240 more bottles.

Example:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel? Some typical estimation strategies for this problem:

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

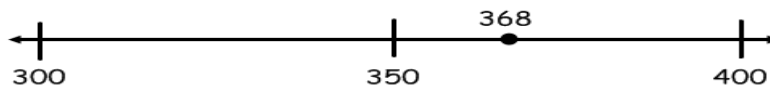Major   Supporting   Additional   Depth Opportunities(DO)

Example:

Round 368 to the nearest hundred.

This will either be 300 or 400, since those are the two hundreds before and after 368.

Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400.

Since 368 is closer to 400, this number should be rounded to 400

When students are asked to round large numbers, they first need to identify which digit is in the appropriate place.

Example or reasoning:

Round 76,398 to the nearest 1000.
- Step 1: Since I need to round to the nearest 1000, then the answer is either 76,000 or 77,000.
- Step 2: I know that the halfway point between these two numbers is 76,500.
- Step 3: I see that 76,398 is between 76,000 and 76,500.
- Step 4: Therefore, the rounded number would be 76,000.

Instructional Strategies: See Grade 4.NBT.1

Tools/Resources:

4.NBT Rounding to the Nearest 1000
3.NBT, 4.NBT Rounding to the Nearest 100 and 1000

Common Misconceptions: See Grade 4. NBT.1

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding to perform multi-digit arithmetic.*

Standard: Grade 4.NBT.4

Fluently add and subtract multi-digit whole numbers using the standard algorithm**.**

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.5   Use appropriate tools strategically.
- ✓ MP.7   Look for and make use of structure.
- ✓ MP.8   Look for and express regularity in repeated reasoning.

Connections: (4.NBT.4-6)

This Cluster is connected to:

- Fourth Grade Critical Areas of Focus #1 , Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends, and go beyond to address adding and subtracting multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 3 NBT 2 – 3)
- Use the four operations with whole numbers to solve problems (Grade 4 OA 2 – 3).
- Generalize place value understanding for multi-digit whole numbers (Grade 4 NBT 1 – 2).

Explanation and Examples:

Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to **fluency, which means accuracy and efficiency (using a reasonable amount of steps and time), and flexibility (using a variety of strategies such as the distributive property, decomposing and recomposing numbers, etc.).**

[Kansas State Department of Education White Paper on Fluency](#)

This is the first-grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

Major   Supporting   Additional   Depth Opportunities(DO)

3892
+1567

Student explanation for this problem:
1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the 10 tens as one more hundred.(notates with a 1 above the hundreds column)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

3546
- 928

Student explanation for this problem:
1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.)
2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)
3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it.) (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).
6. I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)

Note: Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

Instructional Strategies: (4.NBT.4-6)
A crucial theme in multi-digit arithmetic is encouraging students to develop *strategies* that they understand, can explain, and can think about, rather than merely follow a sequence of directions, rules or procedures that they don't understand. It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to *understand* all the steps in the algorithm, and they should be able to explain the meaning of each digit.

For example, a 1 can represent one, ten, or hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Major   Supporting   Additional   Depth Opportunities(DO)

Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers be sure to provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even *attempt* to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (multiples of 10) when doing mental math, and this promotes its understanding.

Tools/Resources
See engageNY Math Module for NBT.4:
https://www.engageny.org/resource/grade-4-mathematics-module-1

See also: "Grocery Shopping, Georgia Department of Education. This task provides students with the opportunity to apply estimation strategies and an understanding of how estimation can be used as a real life application. For this activity, it is expected that students have been introduced to rounding as a process for estimating.

Common Misconceptions: (4.NBT.4-6)
Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

Specific strategies or students having difficulty with lining up similar place values in numbers as they are adding and subtracting.

Sometimes it is helpful to have them write their calculations on grid paper or lined notebook paper with the lines running vertical. This assists the student with lining up the numbers more accurately.

Major   Supporting   Additional   Depth Opportunities(DO)

If students are having a difficult time with a standard addition algorithm, a possible modification to the algorithm might be helpful. Instead of the 'shorthand' of 'carrying,' students could add by place value, moving left to right placing the answers down below the 'equals' line. For example:

$$
\begin{array}{r}
249 \\
372 \\
500 \\
110 \\
+\ 11 \\
\hline
621
\end{array}
$$

(start with 200 + 300 to get the 500, then 40 + 70 to get 110, and 9 + 2 for 11)

Domain: Number and Operations in Base Ten (NBT)

Standard: Grade 4.NBT.5

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NBT.4

Explanation and Examples:

Students who develop flexibility in breaking numbers apart (decomposing numbers) have a better understanding of the importance of place value and the distributive property in multi-digit multiplication.

Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication and understanding why it works, is an expectation in the 5th grade.

This standard calls for students to multiply numbers using a variety of strategies.

Example:

There are 25 dozen cookies in the bakery. What is the total number of cookies at the baker?

Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 and 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 50 x 6 = 300

Major   Supporting   Additional   Depth Opportunities(DO)

Use of place value and the distributive property are applied in the scaffold examples below.

- To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 X 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.
- The area model shows the partial products.

14 x 16 = 224

100 + 40 + 60 + 24 = 224
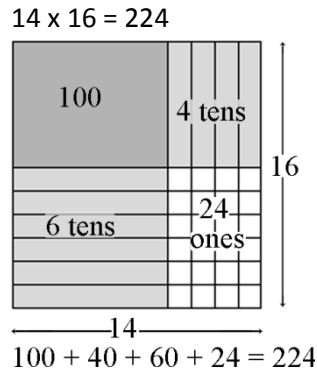
Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

- Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

- Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

```
    25
X 24
400 (20 x 20)
100 (20 x 5)
 80 (4 x 20)
 20 (4 x 5)
600
```

-
```
    25
X 24
500 (20 x 25)
100 (4 x 25)
600
```

- **Matrix Model:** This model should be introduced after students have facility with the strategies shown above.

	20	5	
20	400	100	500
4	80	20	100
	480 +	120	600

Example:

What would an array area model of 74 X 38 look like?

	70	4
30	$70 \times 30 = 2,100$	$4 \times 30 = 120$
8	$70 \times 8 = 560$	$4 \times 8 = 32$

$2,000 = 560 + 1,200 + 32 = 2,812$

Instructional Strategies: See Grade 4.NBT.4

Tools/Resources

4.NBT Thousands and Millions of Fourth Graders

See also: "Using Arrows to Multiply Bigger Numbers", Georgia Department of Education. In this task students demonstrate how to multiply two-digit numbers using arrays. Students will be given a multiplication problem with a two-digit number by a two-digit number. They will use graph paper to solve the problem by breaking it down into partial products (smaller arrays to find the answer).

For detailed information see Progressions for the Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten

Common Misconceptions: See Grade 4.NBT.4

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding a properties of operations to perform multi-digit operations.*

Standard: Grade 4.NBT 6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models**.**

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NBT.4

Explanation and Examples:
In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Examples:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- **Using Base 10 Blocks**: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** $260 \div 4 = (200 \div 4) + (60 \div 4)$
- **Using Multiplication:** $4 \times 50 = 200$, $4 \times 10 = 40$, $4 \times 5 = 20$; $50 + 10 + 5 = 65$; so $260 \div 4 = 65$

This standard calls for students to explore division through various strategies.

Student 1	Student 2			Student 3
592 divided by 8	592 divided by 8			I want to get to 592
There are 70 8's in 560	I know that 10 8's is 80	592		8 x 25 = 200
592 - 560 = 32	If I take out 50 8's that is 400	-400	50	8 x 25 = 200
There are 4 8's in 32	592 - 400 = 192	192		8 x 25 = 200
70 + 4 = 74	I can take out 20 more 8's which is 160	- 160	20	200 + 200 + 200 = 600
	192 - 160 = 32	32		600 - 8 = 592
	8 goes into 32 4 times	-32	4	I had 75 groups of 8 and
	I have none left	0		took one away, so there are
	I took out 50, then 20 more,			74 teams
	then 4 more			
	That's 74			

Example:

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

$$150 \div 6$$

	6			6	
			10	60	
	150	→	10	60	
			5	30	
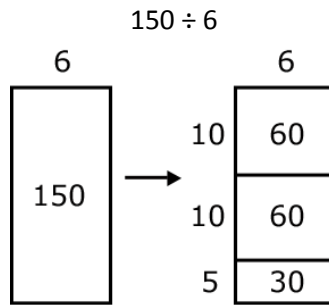
Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:

    a.

$$\begin{array}{r} 150 \\ -60\,(6 \times 10) \\ \hline 90 \\ -60\,(6 \times 10) \\ \hline 30 \\ -30\,(6 \times 5) \\ \hline 0 \end{array}$$

$$150 \div 6 = 10 + 10 + 5 = 25$$

    b. $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

Example:

1917×9

9
1800
90
←27
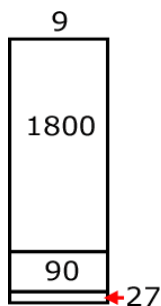
A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines.

$$1917 \div 9 = 213$$

Instructional Strategies: See Grade 4.NBT. 4

Tools/Resources

For detailed information see [Progressions for the Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten](#)

[4.NBT Mental Division Strategy](#)

Common Misconceptions: See Grade 4.NBT. 4

Domain: Number and Operations – Factions (NF)

Cluster: *Extend operations of fraction equivalency and ordering.*

Standard: Grade 4.NF.1

Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (4.NF.1-2)

This cluster is connected to:
- Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**
- Develop understanding of fractions as numbers (Grade 3 NF 3).

Explanation and Examples:

This standard refers to visual fraction models. This includes area models, linear models (number lines) or it could be a collection/set models.

This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).
Students can use visual models or applets to generate equivalent fractions.

Example:

All the area models show $\frac{1}{2}$. The second model shows $\frac{2}{4}$ but also shows that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions because their areas are equivalent. When a horizontal line is drawn through the center of the model, the number of equal parts doubles and size of the parts is halved.

Students will begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

$$\frac{1}{2}$$

$$\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$$

$$\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$$

$$\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$$

Major   Supporting   Additional   Depth Opportunities(DO)
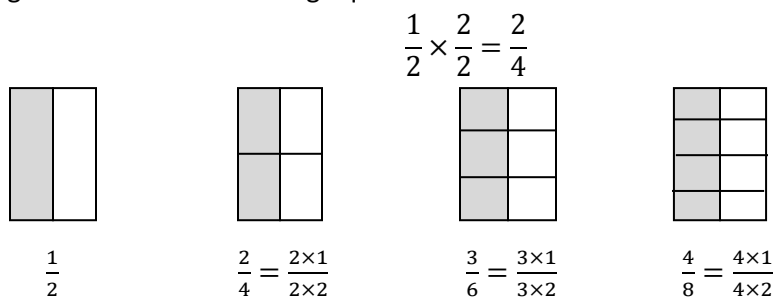
Instructional Strategies: **(4.NF.1-2)**

Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions.

Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators.

Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole.

The models should be represented in drawings. Students should also use **benchmark fractions** such as $\frac{1}{2}$ to compare two fractions and explain their reasoning. The result of the comparisons should be recorded using >, < and = symbols.

Example of Conceptual Reasoning: "I know that 5/8 is a little bit more (1/8 more) than the benchmark 1/2, because 4/8 is equal to 1/2, so I would place 5/8 just to the right of 1/2 on the number line".

Tools/Resources

For Additional Information See Progressions for the Common Core State Standards in Mathematics: 3-5 Number and Operations-Fractions

4.NF Money in the piggy bank
4.NF Running Laps
4.NF Explaining Fraction Equivalence with Pictures
4.NF Fractions and Rectangles

This technology connection is an activity to create equivalent fractions by dividing shapes and matching them to number line locations.
See Illuminations Equivalent Fractions

Common Misconceptions: (4.NF.1-2)

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths.

They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the "whole fraction".

It's important that students use a fraction in the form of **one** such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.

Domain: Number and Operations- Fractions (NF)

Standard: Grade 4.NF.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NF.1

Explanation and Examples:
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $\frac{15}{9} = \frac{5}{3}$), and they develop methods for generating and recognizing equivalent fractions and can represent equivalent fractions concretely and/or pictorially.

This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. **Students' experiences should focus on visual fraction models rather than algorithms**.

When tested, models may or may not be included. Students should learn to draw fraction models to help them compare and use reasoning skills based on fraction benchmarks.

Students must also recognize that they must consider the size of the whole when comparing fractions (ie, $\frac{1}{2}$ *and* $\frac{1}{8}$ of two medium pizzas is very different from $\frac{1}{2}$ of one medium and $\frac{1}{8}$ of one large). Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Example:

Use patterns blocks.

1. If a red trapezoid is one whole, which block shows $\frac{1}{3}$?

2. If the blue rhombus is $\frac{1}{3}$, which block shows one whole?

3. If the red trapezoid is one whole, which block shows $\frac{2}{3}$?

Melisa used a 12 x 12 grid to represent 1 and Nancy used a 10 x 10 grid to represent 1. Each girl shaded grid squares to show $\frac{1}{4}$. How many grid squares did Melisa shade? How many grid squares did Nancy shade? Why did they need to shade different numbers of grid squares?

Possible solution: Melisa shaded 36 grid squares; Nancy shaded 25 grid squares. The total number of little squares is different in the two grids, so $\frac{1}{3}$ of each total number is different.

Melisa's grid   Nancy's grid
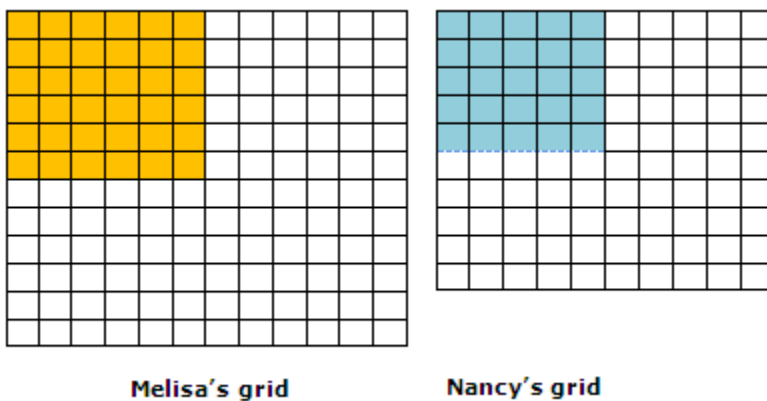
Example:

There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

Student 1	Student 2	Student 3:
Area model: The first cake has more left over. The second cake has $\frac{5}{12}$ left which is smaller than $\frac{1}{2}$.	**Linear/Number Line model:** First Cake $\frac{1}{2}$ Second Cake $0 \quad \frac{3}{12} \quad \frac{6}{12} \quad \frac{9}{12}$	I know that $\frac{6}{12}$ equals $\frac{1}{2}$. Therefore, the second cake which has $\frac{7}{12}$ left is greater than $\frac{1}{2}$. **Benchmark fractions** include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths.

Major   Supporting   Additional   Depth Opportunities(DO)

Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, >, =.

It is important that students explain the relationship between the numerator and the denominator, using Benchmark Fractions. See examples below:

Fractions may be compared using $\frac{1}{2}$ as a benchmark.
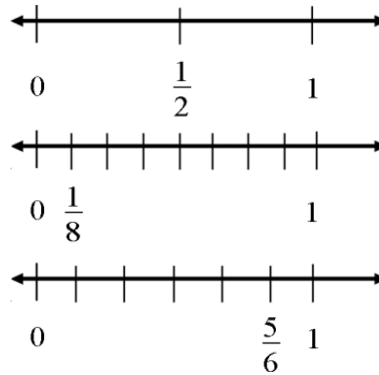
Possible student thinking by using benchmarks:

$\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

Possible student thinking by creating common denominators:

$$\frac{5}{6} > \frac{1}{2} \text{ because } \frac{3}{6} = \frac{1}{2} \text{ and } \frac{5}{6} > \frac{3}{6}$$

Fractions with common denominators may be compared using the numerators as a guide.

$$\frac{2}{6} < \frac{3}{6} < \frac{5}{6}$$

Fractions with common numerators may be compared and ordered using the denominators as a guide.

$$\frac{1}{10} < \frac{3}{8} < \frac{3}{4}$$

Instructional Strategies:

Tools/Resources: See Also Grade 4. NF.1
4.NF Doubling Numerators and Denominators
4.NF Listing fractions in increasing size
4.NF Using Benchmarks to Compare Fractions

Common Misconceptions: See Grade 4. NF.1

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations – Fractions (NF)

Cluster: *Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.*

Standard: Grade 4.NF.3

Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$.

    a.   Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

    b.   Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

         Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}; \frac{3}{8} = \frac{1}{8} + \frac{2}{8}; 2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}.$

    c.   Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

    d.   Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (4.NF.3-4)

This cluster is connected to:

- Fourth Grade Critical Area of Focus #2. **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**
- Represent and interpret data (Grade 4.MD.4)

Explanation and Examples:

A fraction with a numerator of one is called a **unit fraction.** When students investigate fractions other than unit fractions, such as $\frac{2}{3}$, they should be able to decompose the non-unit fraction into a combination of several unit fractions.

Major   Supporting   Additional   Depth Opportunities(DO)

Example:

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example:

$$1\frac{1}{4} - \frac{3}{4} = \square$$

$$\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

$$\frac{5}{4} - \frac{3}{4} = \frac{2}{4} \; or \; \frac{1}{2}$$

Example of word problem:

Mary and Lacey decide to share a pizza. Mary ate $\frac{3}{6}$ and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat together?

Solution: The amount of pizza Mary ate can be thought of a $\frac{3}{6}$ or $\frac{1}{6}$ and $\frac{1}{6}$ and $\frac{1}{6}$. The amount of pizza Lacey ate can be thought of a $\frac{1}{6}$ and $\frac{1}{6}$. The total amount of pizza they ate is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or $\frac{5}{6}$ of the whole pizza.

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:

Susan and Avery need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Avery has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Avery has to find out how much ribbon they have altogether.

Susan has $3\frac{1}{8}$ feet of ribbon and Avery has $5\frac{3}{8}$ feet of ribbon. I can write this as $3\frac{1}{8} + 5\frac{3}{8}$. I know they have 8 feet of ribbon by adding the 3 and 5. They also have $\frac{1}{8}$ and $\frac{3}{8}$ which makes a total of $\frac{4}{8}$ more. Altogether they have $8\frac{4}{8}$ feet of ribbon. $8\frac{4}{8}$ is larger than $8\frac{3}{8}$ so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, $\frac{1}{8}$ foot.

Major　　　　　**Supporting**　　　　　**Additional**　　　　　**Depth Opportunities(DO)**

Example:

Timothy has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{1}{8}$ of a pizza left. How much pizza did Timothy give to his friend?

Solution: Timothy had $4\frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x's show the pizza he has left which is $2\frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is $\frac{13}{8}$ or $1\frac{5}{8}$ pizzas.
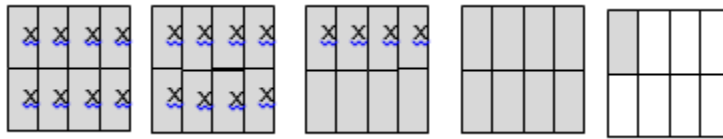
Mixed numbers are introduced for the first time in Fourth Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions. Keep in mind **Concrete-Representation-Abstract (CRA)** approach to teaching fractions. Students need to be able to "show" their thinking using concrete and/or representations BEFORE they move to abstract thinking.

Example:

While solving the problem $3\frac{3}{4} = 2\frac{1}{4}$ students could do the following:

Student 1	Student 2	Student 3
$3 + 2 = 5 \; and \; \frac{3}{4} + \frac{1}{4} = 1$	$3\frac{3}{4} + 2 = 5\frac{3}{4} + \frac{1}{4} = 6$	$3\frac{3}{4} = \frac{15}{4} \; and \; 2\frac{1}{4} = \frac{9}{4} \; so \; \frac{15}{4} + \frac{9}{4} + \frac{24}{4} = 6$

Example:

A cake recipe calls for you to use ¾ cup of milk, $\frac{1}{4}$ cup of oil, and $\frac{2}{4}$ cup of water. How much liquid was needed to make the cake? Use an area model to solve.

$\frac{6}{4} \; or \; 1$ whole cup and $\frac{1}{2}$ more
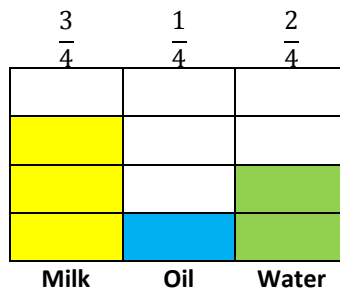
Instructional Strategies:

In Grade 3, students added unit fractions with the same denominator. Now, they begin to represent a fraction by decomposing the fraction as the sum of unit fraction and justify with a fraction model. For example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
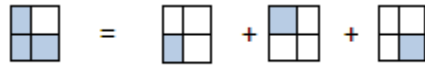
Students also represented whole numbers as fractions. They use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem.

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawings to show their understanding.

Tools/Resources:

4.NF Comparing two different pizzas
4.NF Comparing Sums of Unit Fractions
4.NF Making 22 Seventeenths in Different Ways
4.NF Writing a Mixed Number as an Equivalent Fraction
4.NF Peaches
4.NF Plastic Building Blocks
4.NF Cynthia's Perfect Punch
4.NF Sugar in six cans of soda

See: "Harry's Hike", NCSM, Great Tasks for Mathematics K-5, (2013). Students determine if a given estimate is justifiable and work thorough different forms of modeling to prove or disprove their original hypothesis.

For Additional Information See Progressions for the Common Core State Standards in Mathematics: 3-5 Number and Operations-Fractions

Common Misconceptions: (4.NB.3-4)

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations (NF)

Cluster: *Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.*

Standard: Grade 4.NF.4

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.

b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \frac{a}{b} = \frac{n \times a}{b}$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 4.NF3

Explanation and Examples:

Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns. This standard builds on students' work of adding fractions and extending that work into multiplication. (4.NF.4a)

Major   Supporting   Additional   Depth Opportunities(DO)

Examples:

$$3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$$
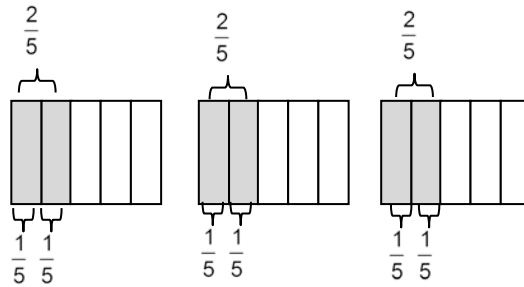
If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?
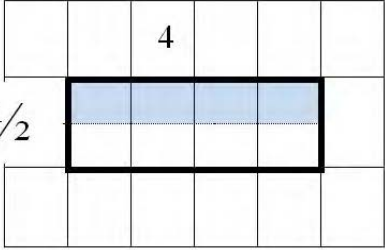
A student may build a fraction model to represent this problem:

$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{15}{8} = 1\frac{7}{8}$$
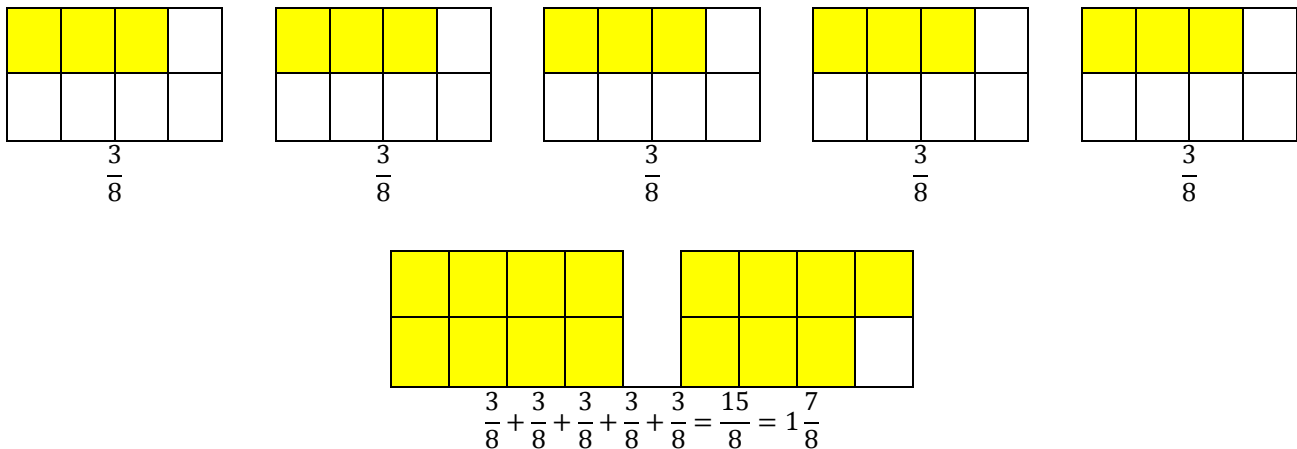
This standard extends the idea of multiplication as repeated addition (4.NF.4b)

For example, $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$. Students are expected to use and create visual fraction models to multiply a whole number by a fraction.

This standard calls for students to use visual fraction models (Area, Linear and Set Models) to solve word problems related to multiplying a whole number by a fraction. (4.NF.4c)

Major   Supporting   Additional   Depth Opportunities(DO)

Student 1	Student 2	Student 3
Draws a number line to show 4 jumps of $\frac{1}{2}$.	Draws and area model showing 4 pieces of $\frac{1}{2}$ joined together to equal 2.	Draws an area model representing $4 \times \frac{1}{2}$ on a grid, dividing each row into $\frac{1}{2}$ to represent the multiplier.

Instructional Strategies: See Grade 4. NF.3

Tools/Resources :
Fraction Tiles, Fraction bars, (Area Models) Rulers, and Number Lines (Linear Models)

[4.NF Sugar in six cans of soda](#)

For Additional Information See [Progressions for the Common Core State Standards in Mathematics: 3-5 Number and Operations-Fractions](#)

Common Misconceptions: See Grade 4. NF.3

Domain: Number and Operations – Fractions (NF)

Cluster: *Understand decimal notation for fractions, and compare decimal fractions.*

Standard: Grade 4.NF.5

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: 4.NF.5-7

This cluster is connected to:

- Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**
- Connect with understanding and generating equivalent fractions (Grade 4 NF 1 – 2).
- Students will perform operations with decimals to hundredths in Grade 5 (Grade 5 NBT 5 -7).

Explanation and Examples:

Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work.

Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.

Ones	.	Tenths	Hundreths

Tenths Grid .3 = 3 tenths = 3/10   Hundreth Grid .30 = 30 hundreths = 30/100
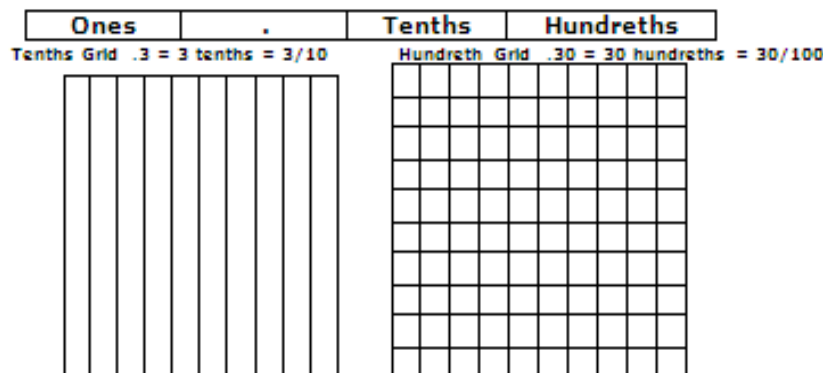
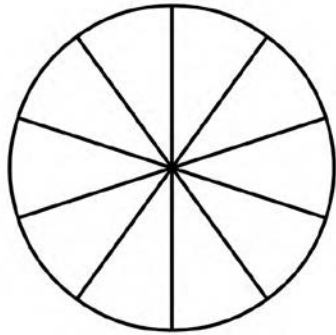
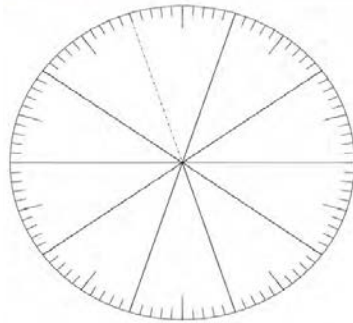Major   Supporting   Additional   Depth Opportunities(DO)

Example:

Represent 3 tenths and 30 hundredths on the models shown below:

10ths circle   **100ths circles**

Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.
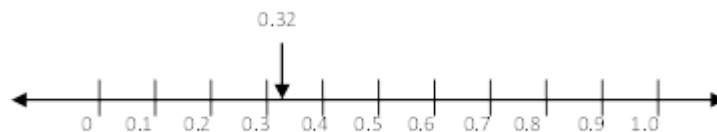
Base Ten Blocks: students may represent $\frac{3}{10}$ with 3 longs and may also write the fraction as $\frac{30}{100}$ with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	.	Tenths	Hundredths
			.	3	2

Students use the representations explored in 4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$.

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.

0.32

```
    0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0
```

Instructional Strategies:

Students extend fraction equivalence from Grade 3 with denominators of 2, 3 4, 6 and 8 to fractions with a denominator of 10. Provide fraction models of tenths and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

The place value system developed for whole numbers extends to fractional parts represented as decimals. This is a connection to the metric system.

Decimals are another way to write fractions. The place-value system developed for whole numbers extends to decimals. The concept of one whole used in fractions is extended to models of decimals.

As mentioned above, students can use base-ten blocks to represent decimals. A 10 x 10 block can be assigned the value of one whole to allow other blocks to represent tenths and hundredths. They can show a decimal representation from the base-ten blocks by shading on a 10 x 10 grid.

It is important that students make connections between fractions and decimals. They should be able to write decimals for fractions with denominators of 10 or 100. Have students say the fraction with denominators of 10 and 100 aloud. For example $\frac{4}{10}$ would be "four tenths" or $\frac{27}{100}$ would be "twenty-seven hundredths." Also, have students represent decimals in word form and the decimal place value.

Students should be able to express decimals to the hundredths as the sum of two decimals or fractions. This is based on understanding of decimal place value. For example 0.32 would be the sum of 3 tenths and 2 hundredths. Using this understanding students can write 0.32 as the sum of two fractions ($\frac{3}{10} + \frac{2}{100}$)

Students' understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5.

In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations before they try to do conversions in the metric system.

Understanding of the decimal place value system is important **prior** to the generalization of moving the decimal point when performing operations involving decimals.

When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 x 10 grid. The 10 x 10 grid is defined as one whole. The decimal must relate to the whole.

0.3   0.03
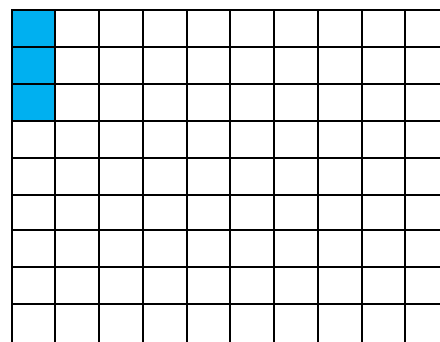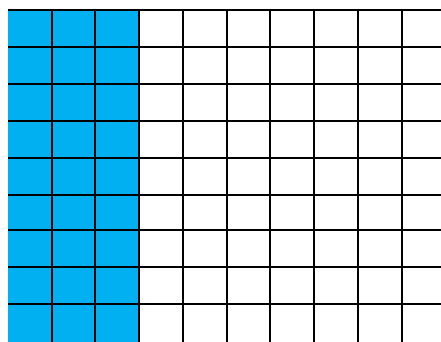
Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

Major   Supporting   Additional   Depth Opportunities(DO)

Resources/Tools

Length or area models

10 x 10 square on a grid

Decimal place-value mats

Base-ten blocks

Number lines

4.NF Expanded Fractions and Decimals

4.NF Dimes and Pennies

4.NF Fraction Equivalence

4.NF How Many Tenths and Hundredths?

4.NF Adding Tenths and Hundredths

"A Meter of Candy", NCTM.org, Illuminations–This series of three hands-on activities for students to develop and reinforce their understanding of hundredths as fractions, decimals and percentages. Students explore using candy pieces and make physically models to connect a set and linear model (meter); produce area models (grids and pie graphs).

"Flag Fractions", Georgia Department of Education.   Students create a flag by coloring fractional pieces of the flag and name and write the fractional parts created on their flag. While exploring, students add decimal fractions with like denominators, write decimal fractions as decimals, order two digit decimals, and add two digit decimals.

Common Misconceptions: 4.NF.5-7

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that .03 is greater than 0.3.

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Number and Operations – Fractions (NF)

Standard: Grade 4.NF.6

Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.NF.5

Explanation and Examples:

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	●	Tenths	Hundredths
			●	3	2

Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100.

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.

Instructional Strategies: See Grade 4.NF.5

Tools/Resources

4.NF Expanded Fractions and Decimals
4.NF Dimes and Pennies
4.NF How Many Tenths and Hundredths?

Common Misconceptions: See Grade 4.NF.5

Major	Supporting	Additional	Depth Opportunities(DO)

Domain: Number and Operations – Fractions (NF)

Cluster: *Understand decimal notation for fractions, and compare decimals fractions.*

Standard: Grade 4.NF.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.4   Model with mathematics.
- ✓ MP.5   Use appropriate tools strategically.
- ✓ MP.7   Look for and make use of structure.

Connections: See Grade 4. NF.5

Explanation and Examples:
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language.

The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, ‹, ›, =**

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases.

Each of the models below shows $\frac{3}{10}$ but the whole on the right is much bigger than the whole on the left. They are both $\frac{3}{10}$ but the model on the right is a much larger quantity than the model on the left.
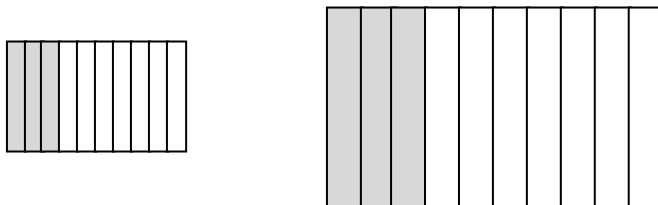
When the wholes are the same, the decimals or fractions can be compared.
Example:
Draw a model to show that 0.3 < 0.5. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.

Major   Supporting   Additional   Depth Opportunities(DO)

Instructional Strategies: See Grade 4. NF.5

Tool/Resources

4.NF Using Place Value

"Flag Fractions", Georgia Department of Education.   Students create a flag by coloring fractional pieces of the flag and name and write the fractional parts created on their flag. While exploring, students add decimal fractions with like denominators, write decimal fractions as decimals, order two digit decimals, and add two digit decimals.

Common Misconceptions: See Grade 4. NF.5

Domain: Measurement and Data (MD)

Cluster: *Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.*

Standard: Grade 4.MD.1

Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1   Solve problems and persevere in solving them
- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.5   Use appropriate tools strategically.
- ✓ MP.6   Attend to precision.

Connections: 4.MD.1-3

This cluster is connected to:

- Fourth Grade Critical Areas of Focus #1, **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends**, and #2, **Developing an understanding of fractions equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers**.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures (Grade 3 MD 8).
- Geometric measurement; understand concepts of area and relate area to multiplication and to addition. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (Grade 4 NF 3 – 4).

Explanation and Examples:

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter.**

The units of measure that have not been addressed in prior years are pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass, liquid volume, and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure.

Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.

Major   Supporting   Additional   Depth Opportunities(DO)

Example:

kg	g
1	1000
2	2000
3	3000

ft	in
1	12
2	24
3	36

lb	oz
1	16
2	32
3	48

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (**partition**).
- Understand that the same unit can be repeated to determine the measure (**iteration**).
- Understand the relationship between the size of a unit and the number of units needed (**compensatory principal**).

Instructional Strategies:

In order for students to have a better understanding of the **relationships** between units, they need to use measuring devices in class.

The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard.

Allow students to use rulers or a yardstick to discover these **relationships** among these units of measurements. Using 12-inch rulers and yardstick, students can see that three of the 12-inch rulers, is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick.

Have students record the relationships in a two column table or t-charts. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters.

Present word problems as a source of students' understanding of the relationships among inches, feet and yards.

Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Present problems that involve multiplication of a fraction by a whole number (denominators are 2, 3, 4, 5 6, 8, 10, 12 and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts.

Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems.

Resources/Tools

For detailed information see Learning Progressions for Measurement:

http://commoncoretools.files.wordpress.com/2012/07/ccss_progression_gm_k5_2012_07_21.pdf

Yardsticks(meter sticks) and rulers (marked with customary and metric units)

Major   Supporting   Additional   Depth Opportunities(DO)

Teaspoons and tablespoons
Graduated measuring cups (marked with customary and metric units)

"Kilogram Scavenger Hunt", Georgia Department of Education. Students will items in the classroom that they think weighs about a kilogram. They will actually weigh the items to see how close their estimates were to the exact weight.

Common Misconceptions: 4.MD.1-3
Students believe that larger units will give the larger measure.

Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yardsticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles and explain their reasoning.

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Measurement and Data (MD)

Cluster: *Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit*

Standard: Grade 4.MD.2

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See Grade 4.MD.1

Explanation and Examples:

This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents).

Students should have ample opportunities to use number line diagrams to solve word problems.

Example:

Debbie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?

Possible solution: Debbie plus 10 friends = 11 total people

11 people X 8 ounces (glass of milk) = 88 total ounces

1 quart = 2 pints = 4 cups = 32 ounces

Therefore 1 quart = 2 pints = 4 cups = 32 ounces

2 quarts = 4 pints = 8 cups = 64 ounces

3 quarts = 6 pints = 12 cups = 96 ounces

If Debbie purchased 3 quarts (6 pints) of milk there would be enough for everyone at her party to have at least one glass of milk. If each person drank 1 glass then she would have 1- 8 oz glass or 1 cup of milk left over.

Examples with various operations:

Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?

Major   Supporting   Additional   Depth Opportunities(DO)

Students may record their solutions using fractions or inches. (The answer would be $\frac{2}{3}$ of a foot or 8 inches.

Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.)

Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.

Example:
At 7:00 a.m. Melisa wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.

Instructional Strategies: See Grade 4.MD.1

Resources/Tools
4.MD Margie Buys Apples

Common Misconceptions: See Grade 4.MD.1

Domain: Measurement and Data (MD)

Cluster: *Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.*

Standard: Grade 4.MD.3

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 4.MD.1

Explanation and Examples:

Students developed understanding of area and perimeter in 3rd grade by using **visual** models. While students are expected to use formulas to calculate area and perimeter of rectangles, they need to **understand and be able to communicate their understanding of why the formulas work**.

The formula for area is $l \times w$ and the answer will always be in square units. The formula for perimeter can be $2l + 2w$ *or* $2(l + w)$ and the answer will be in linear units.

This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:

Mrs. Fields is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will she need to cover the entire course?

1 square foot of carpet
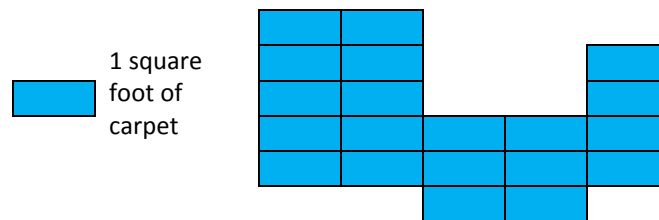
Instructional Strategies: See Grade 4.MD.1

Resources/Tools

[4.OA, MD Karl's Garden](#)

Common Misconceptions: See Grade 4.MD.1

Domain: Measurement and Data (MD)

Standard: Grade 4.MD.4

Make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:
- Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**
- Understand a fraction as a number on the number line; represent fractions on a number line diagram (Grade 3 NF 2).
- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size (Grade 3 NF 3).
- Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters (Grade 3 MD 4).
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem (Grade 4 NF 3d).

Explanation and Examples:

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **data, line plot, length, fractions.**

This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:

Students measured objects in their desk to the nearest $\frac{1}{2}, \frac{1}{4}$ *or* $\frac{1}{8}$ inch. They displayed their data collected on a line plot. How many object measured $\frac{1}{4}$ inch? $\frac{1}{2}$ inch? If you put all the objects together end to end what would be the total length of **all** the objects.
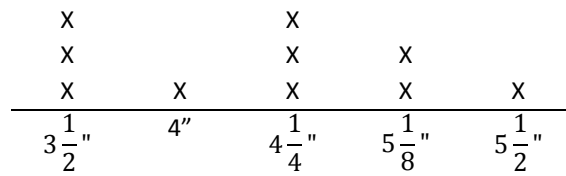
Ten students in Room 31 measured their pencils at the end of the day. They recorded their results on the line plot below.

```
        X                   X
        X                   X       X
        X       X           X       X       X
      ─────────────────────────────────────────
        1       4"          1       1       1
       3─ "               4─ "     5─ "     5─ "
        2                   4       8       2
```

Possible questions:
- What is the difference in length from the longest to the shortest pencil?
- If you were to line up all the pencils, what would the total length be?
- If the $5\frac{1}{8}$" pencils are placed end to end, what would be their total length?

Instructional Strategies:
Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit ($\frac{1}{2}, \frac{1}{4}$ or $\frac{1}{8}$) and plot data showing multiple data points for each fraction.
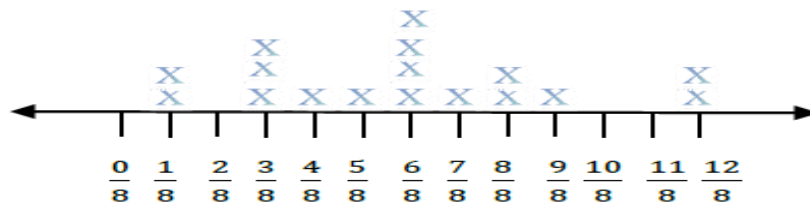
Pose questions that students may answer, such as:
- "How many one-eighths are shown on the line plot?" Expect "two one-eighths" as the answer.
- Then ask, "What is the total of these two one-eighths?" Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.
- "What is the total number of inches for insects measuring $\frac{3}{8}$ inches?" Students can use skip counting with fraction names to find the total, such as, "three-eighths, six-eighths, nine-eighths. The last fraction names the total.
- Students should notice that the denominator did not change when they were saying the fraction name.
- Have them make a statement about the result of adding fractions with the same denominator.
- "What is the total number of insects measuring $\frac{1}{8}$ inch or $\frac{5}{8}$ inches?" Have students write number sentences to represent the problem and solution such as, $\frac{1}{8} + \frac{1}{8} + \frac{5}{8} = \frac{7}{8}$ inches.

Use visual fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

Resources/Tools

For detailed information see Learning Progression for Data:
http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf

4.MD, 5.MD Button Diameters

See engageNY Module 5, Topic E, F, & G for Lessons:
file:///C:/Users/Melisa/Downloads/math-g4-m5-full-module.pdf

See also : "Bugs, Giraffes, Elephants, and More", NCSM, Great Tasks for Mathematics K-5. (2013). Students interpret line plots with scales written to the nearest quarter of a unit.

Fraction bars or strips
Number Lines

Common Misconceptions:

Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

Students also count the tick marks on the number line to determine the fraction, rather than looking at the "distance" or "space" between the marks.

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Measurement and Data (MD)

Standard: Grade 4.MD.5

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.

   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees**.**

Suggested Standards for Mathematical Practice (MP):

   ✓ MP.5   Use appropriate tools strategically.
   ✓ MP.6   Attend to precision.
   ✓ MP.7   Look for and make use of structure.

Connections: 4.MD.5-7

This cluster is connected to:

   • Fourth Grade Critical Area of Focus #3, **Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.**
   • Connect measuring angle to the Geometry domain in which students draw and identify angles as right, acute and obtuse (Grade 4. G. 1).

Explanation and Examples:

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown**

This standard calls for students to explore the connection between angles (measure of rotation) and circular measurement (360 degrees).

The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles, yet the angle measure is the same.
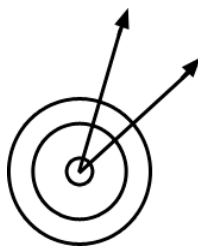
Students explore an angle as a series of "one-degree turns."   A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?

Major   Supporting   Additional   Depth Opportunities(DO)

Instructional Strategies:

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger.

Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.

Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles.
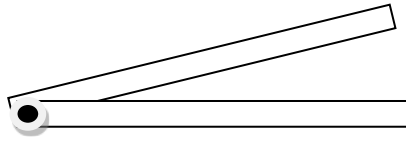
They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.

Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler.

Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays. Each set of straws can be attached with two jointed paper clips.

Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle.

Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a **relationship** between the number of degrees in an angle and circle which has a measure of 360 degrees.

Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Resources/Tools

Cardboard cut in strips to make an angle explorer

Brass fasteners

Protractor

Angle ruler

Goniometers

Straws /paperclips

Transparencies

Angle explorers

See engageNY Module 4

See: Sir Cumference and the Great Knight of Angleland: In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest. With the help of a family heirloom that functions similar to a protractor, he is able to locate the elusive King Lell and restore him to the throne. In gratitude, the king bestows knighthood on Sir Radius.

"What's My Angle, Figure This Challenge #10", FigureThis.org. Students can estimate the measures of the angles between their fingers when they spread out their hand.

Common Misconceptions: MD 5-7

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers.

Students should have multiple experiences estimating and comparing angles to the Benchmark 90° or right angle.

They should explain their reasoning by deciding first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°.

If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

Some protractors have a protective edge along the bottom. Zero degrees begins about $\frac{1}{4}$ of an inch above the bottom edge. Students often to not take this into account and therefore will have in accurate measures of angles.

Domain: Measurement and Data (MD)

Cluster: *Geometric measurement: understand the concept of angle and measure angles.*

Standard: Grade 4.MD.6

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.4   Model with mathematics.
- ✓ MP.5   Use appropriate tools strategically.
- ✓ MP.6   Attend to precision.

Connections: See Grade 4. MD.5

Explanation and Examples:

Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles.

They transfer their understanding that a 360° rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90° and 180°.

They extend this understanding and recognize and sketch angles that measure approximately 45° and 30°. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

Students should estimate angles, measure angles and sketch angles. They need to experience measuring angles using an angle ruler as well as a protractor. (The angle ruler allows them to "see" the turns or rotations).

| *120 degrees* | *135 degrees* |
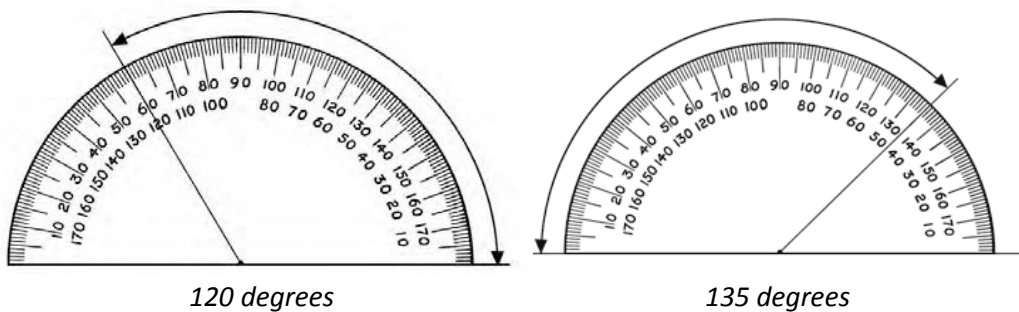
Instructional Strategies: See Grade 4.MD.5

Resources/Tools

4.MD,G Measuring Angles

Common Misconceptions: See Grade 4.MD.5

Domain: Measurement and Data (MD)

Standard: Grade 4.MD.7

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision.

Connections: See Grade 4.MD.5

Explanation and Examples:

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.

Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of m?
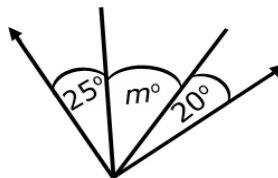
Example:

- Joe Dan knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?
- The five shapes in the diagram are the exact same size. Write an equation that will help you find the measure of the indicated angle. Find the angle measurement.
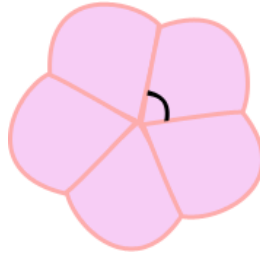
Instructional Strategies: See Grade 4.MD.6

Tools/Resources

4.MD,G Finding an unknown angle
4.MD,G Measuring Angles

Common Misconceptions: See Grade 4. MD.5

Domain: Geometry (G)

Standard: Grade 4.G.1

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Suggested Standards for Mathematical Practice (MP):

✓ MP.5   Use appropriate tools strategically.
✓ MP.6   Attend to precision.

Connections: 4.G.1-3

This cluster is connected to:

- Fourth Grade Critical Area of Focus #3**, Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.**
- Geometric measurement: understand concepts of angles and measure angles (Grade 4 MD 3). Symmetry can be related to experiences in art.

Explanation and Examples:

This standard asks students to draw two-dimensional geometric objects and to also identify them in two- dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines.

Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily.

Students do not easily identify lines and rays because they are more abstract.
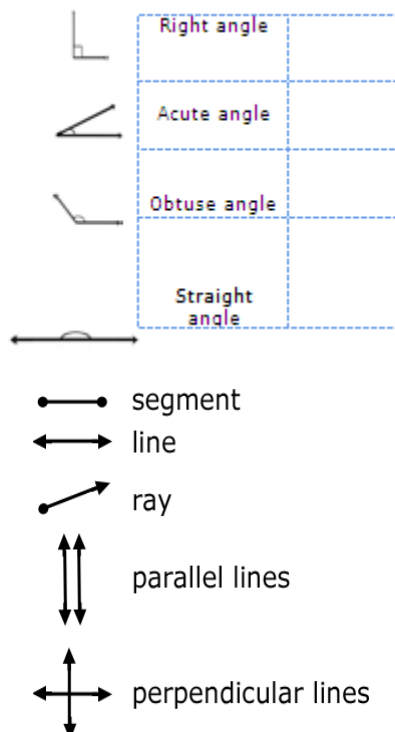
Example:
Draw two different types of quadrilaterals that have two pairs of parallel sides?
Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.
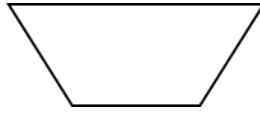
Example:
How many acute, obtuse and right angles are in this shape?
Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

Instructional Strategies:

Angles
Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as greater than, less than, or the same size as a right angle.

Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Symmetry
When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.

With the use of a dynamic geometric program, students can easily construct points, lines and geometric figures. They can also draw lines perpendicular or parallel to other line segments.

Resources/Tools
Mirrors, Miras
Geoboards
GeoGebra is a free dynamic software for learning and teaching

4.G The Geometry of Letters
4.G What's the Point?
4.MD,G Measuring Angles

Common Misconceptions:
Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Geometry (G)

Standard: Grade 4.G.2

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles**.**

Suggested Standards for Mathematical Practice (MP):
- ✓ MP.5   Use appropriate tools strategically.
- ✓ MP.6   Attend to precision.

Connections: See Grade 4.G.1

Explanation and Examples:

Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

Parallel or Perpendicular Lines:

Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90°).

Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.
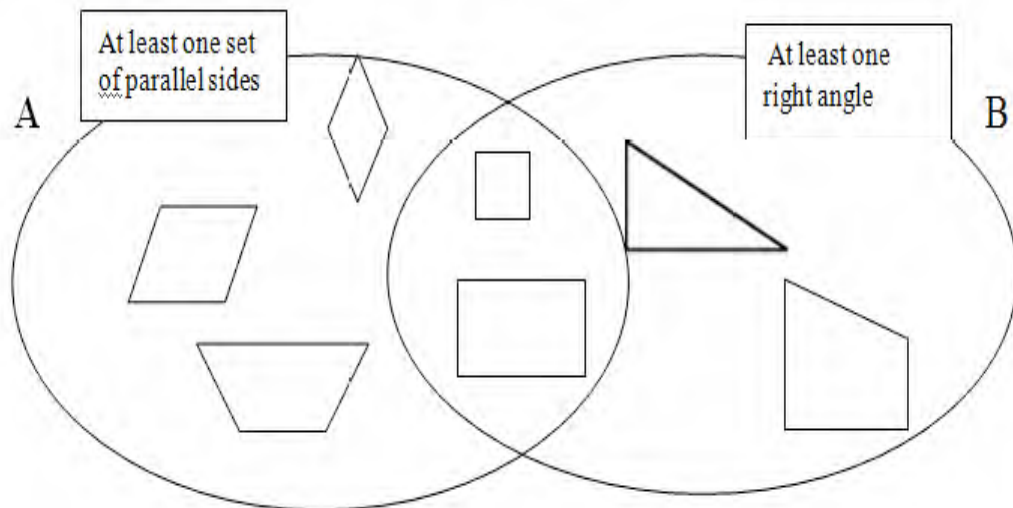
Parallel and perpendicular lines shown below:
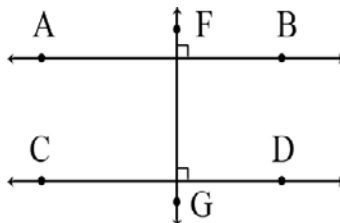
This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.

Example:

Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show counter example.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. *(impossible)*
- Every square is a quadrilateral.
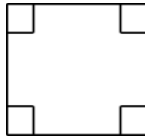- Every trapezoid is a parallelogram. *(impossible)*

Example:
Identify which of these shapes have perpendicular or parallel sides and justify your selection.
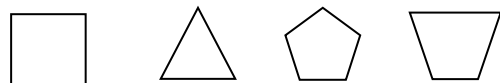
A possible justification that students might give is:

The square has perpendicular lines because the sides meet at a corner, forming right angles.

Angle Measurement:

This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1.

Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.

 Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

Instructional Strategies: **See Grade 4. G.1**

Tools/Resources

4.MD,G Finding an unknown angle
4.G Are these right?
4.G What shape am I?
4.G Defining Attributes of Rectangles and Parallelograms
4.G What is a Trapezoid? (Part 1)

See also: "Quadrilateral Challenge", Georgia Department of Education. Working in pairs, students create the following quadrilaterals. They will identify the attributes of each quadrilateral, then compare and contrast the attributes of different quadrilaterals.

Common Misconceptions: See Grade 4. G.1

Major   Supporting   Additional   Depth Opportunities(DO)

Domain: Geometry (G)

Standard: Grade 4.G.3

Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

**Connections: See Grade 4.G.1**

Explanation and Examples:

Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

This standard only includes line symmetry not rotational symmetry.

Example:

For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions.

Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.
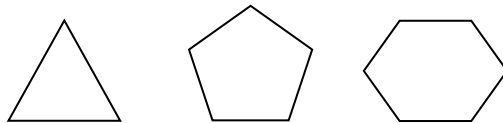
Instructional Strategies:

Give student experience with many shapes that can be folded to determine if they have symmetry. Block letter of the alphabet is one set that students can explore. Students can search magazines to find shapes that are symmetrical and fold to show the line of symmetry.

The use of Miras help students see and draw line to show symmetry. The reflection from the Mira or a mirror helps students see symmetry. Pattern blocks and tangrams are also useful tools in discovering symmetry.

Major   Supporting   Additional   Depth Opportunities(DO)

Resources/Tools

Common Misconceptions:

Some children may think that there can only be one line of symmetry for an object. Encourage them to try folding shapes in more than one way. Giving students multiple copies of the same shapes could help avoid confusion. Coloring one side of the line one color and the other side of the line a different color may aid in seeing multiple lines. In essence the student is seeing if the shape can be folded into $\frac{1}{2}$ halves.

Appendix

TABLE 1. Common Addition and Subtraction Situations[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 =?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2+? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? +3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 =?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5-? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? -2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 =?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3+? = 5 \text{ or } 5 - 3 =?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5 \text{ or } 5 = 5 + 0$ $5 = 1 + 4 \text{ or } 5 = 4 + 1$ $5 = 2 + 3 \text{ or } 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2+? = 5 \text{ or } 5 - 2 =?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 =? \text{ or } 3 + 2 =?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 =? \text{ or } ? +3 = 5$

[1]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common Multiplication and Division Situations[7]

	Unknown Product $3 \times 6 =?$	Group Size Unknown ("How many in each group?" Division) $3 \times? = 18$ and $18 \div 3 =?$	Number of Groups Unknown ("How many groups?" Division) $?\times 6 = 18$ And $18 \div 6 =?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*: You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*: You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b =?$	$a \times? = p$ and $p \div a =?$	$?\times b = p$ and $p \div b =?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The Properties of Operations

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every (a) there exists$(-a)$ so that $a + (-a) = (-a) + a = 0$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

TABLE 4. The Properties of Equality*

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$ then $b = a$
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$
Addition property of equality	If $a = b$ then $a + c = b + c$
Subtraction property of equality	If $a = b$ then $a - c = b - c$
Multiplication property of equality	If $a = b$ then $a \times c = b \times c$
Division property of equality	If $a = b$ and $c \neq 0$ then $a \div c = b \div c$
Substitution property of equality	If $a = b$ then b may be substituted for a in any expression containing a.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.
If $a > b$ and $b > c$ then $a > c$
If $a > b$ then $b < a$
If $a > b$ then $-a < -b$
If $a > b$ then $a \pm c > b \pm c$
If $a > b$ and $c > 0$ then $a \times c > b \times c$
If $a > b$ and $c < 0$ then $a \times c < b \times c$
If $a > b$ and $c > 0$ then $a \div c > b \div c$
If $a > b$ and $c < 0$ then $a \div c < b \div c$

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Table 6. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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