Kansas College Career Ready Standards for Mathematics Flip Book for Grade 1

Updated Fall, 2014

This project used the work done by the Departments of Educations in Ohio, North Carolina, Georgia, engageNY, NCTM, and the Tools for the Common Core Standards.

Major Supporting Additional Depth Opportunities

About the Flip Books

The development of the "flip books" is in response to the adoption of the Common Core State Standards by the state of Kansas in 2010. Teachers who were beginning the transition to the new Kansas Standards—Kansas College and Career Ready Standards (KCCRS) needed a reliable starting place that contained information and examples related to the new standards.

This project attempts to pull together, in one document some of the most valuable resources that help develop the intent, the understanding and the implementation of the KCCRS. The intent of these documents is to provide a starting point for teachers and administrators to begin unraveling the standard and is by no means the only necessary or complete resource that supports implementation of KCCRS.

This project began in the summer 2012 with the work of Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books". The "flip books" are based on a model that Kansas had for earlier standards however, this edition is far more comprehensive than those in the past. The current editions incorporate the resources from: other state departments of education, documents such as the content progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. The current product was a compilation of work from the project developers in conjunction with many mathematics educators from around the state. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KATM website at www.katm.org and will continue to undergo changes periodically. When significant changes/additions are implemented the necessary modification will be posted and dated.

The initial development of the current update to the "flip books" was driven by the need expressed by teachers of mathematics in Kansas and with the financial support from Kansas Department of Education and encouragement from the Kansas Association of Teachers of Mathematics. These "flip books" have become an ongoing resource that will continue to evolve as more is learned about high quality instruction for the KCCRS for mathematics.

For questions or comments about the flipbooks please contact Melisa Hancock at melisa@ksu.edu.

Planning Advice--Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptually understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in "Additional and Sample clusters". Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with "+" within the standards. These standards fall outside of priority recommendations.

Recommendations for using cluster level priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters' headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.

DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus**, **rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. (See Depth of Knowledge (DOK), Table 7, Appendix)

In this document, Depth Opportunities are highlighted pink in the Standards section. For example:

5.NBT.6 *Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.*

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Grade 1

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 2 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 1 examine problems (tasks), can make sense of the meaning of the task and find an entry point or a way to start the task. Grade 1 students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. In Grade 1, students' work still relies on concrete manipulatives and pictorial representations as students solve tasks unless the CCSS refers to the word fluently, which denotes mental mathematics. Grade 1 students also are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. For example, to solve a problem involving multi-digit numbers, they might first consider similar problems that involve multiples of ten. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. They often check their answers to problems using a different method or approach and lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?"
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 1 make sense of quantities and the relationships while solving tasks. This involves two processes- *decontextualizing* and *contextualizing*. In Grade 1, students represent situations by *decontextualizing* tasks into numbers and symbols and *contextualizing* numbers and symbols. For *contextualizing* example, when a student sees the expression 40 – 26, she might visualize this problem by thinking, if I have 26 marbles and Melissa has 40, how many more do I need to have as many as Melissa? Then, in that context, she thinks, 4 more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14. In this example, the student uses a context to think through a strategy for solving the problem, using the relationship between addition and subtraction and decomposing and recomposing the quantities. She then uses what she did in the context to identify the solution of the original abstract problem. A *decontextualizing* example: to find the area of the floor of a rectangular room that measures 10m by 12m, a student might represent the problem as an equation, solve it mentally, and record the problem and solution as $10 \times 12 = 120$.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 1 accurately use definitions and previously established solutions to construct viable arguments about mathematics. In Grade 1 during discussions about problem solving strategies, students constructively critique the strategies and reasoning of their classmates. Examples: 1)while solving $74 + 18 - 37$, students may use a variety of strategies, and after working on the task, can discuss and critique each other's reasoning and strategies, citing similarities and differences between strategies, 2)a student might argue that 2 different shapes have equal area because it has already been demonstrated that both shapes are half of the same rectangle. Students at this level present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written).

Major **Supporting** **Additional** **Depth Opportunities**

4) Model with mathematics.	Mathematically proficient students in Grade 1 model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. At this level it might be as simple as writing an addition equation to describe a situation. Grade 1 students still rely on concrete manipulatives and pictorial representations while solving problems, but the expectation is that they will also write an equation to model problem situations. Likewise, Grade 1 students are expected to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation $24 + 17 - 13 = ?$ *See Table 1 in Appendix for Addition/Subtraction "Situations".*
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 1 have access to and use tools appropriately. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, fractions bars, etc.) drawings or diagrams (number lines, tally marks, tape diagrams, arrays, tables, graphs, etc.), paper & pencil, rulers, and other measuring tools, scissors, tracing paper, grid paper, virtual manipulatives or other available technologies that support conceptual understanding and higher-order thinking skills. Example: while solving $28 + 17$, students can explain why place value blocks are more appropriate than counters.
6) Attend to precision.	Mathematically proficient students in Grade 1 are precise in their communication, calculations, and measurements. They start by using everyday language to express their mathematical ideas, realizing that they need to select words with clarity and specificity rather than saying "it works" without explaining what "it" means. Once Grade 1 students become familiar with a mathematical idea or object, they are ready to learn more precise mathematical terms to describe it. In all mathematical tasks, it is expected that Grade 1 students communicate clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Example: the equivalence of 8 & 5 can be written both as 5 + 3 = 8 and 8 + 5 + 3.
7) Look for and make use of structure.	Mathematically proficient students in Grade 1 carefully look for patterns and structures in the number system and other areas of mathematics. At this level, students USE structure such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference). Or, while solving addition and subtraction problems students can apply the patterns of the number system to skip count by 10s off the decade. For example, Grade 1 students are expected to mentally reason that $33 + 21$ is 33 plus 2 tens, which equals 53 and then an addition one which equals 54. While working in the Numbers in Base Ten domain, students work with the idea that 10 ones equal a ten, and 10 tens equals 1 hundred. Further, Grade 1 students also make use of structure when they work with subtraction as missing addend problems, such as $50 - 33 = ?$ can be written as $33 + ? = 50$ and can be thought of as how much more do I need to add to 33 to get to 50?
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 1 begin to look for regularity in problem structures when solving mathematical tasks. For example, first graders might notice that when tossing 2-color counters to find combinations of a given number, they always get what they call "opposites"----when tossing 6 counters, they get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 10 counters, they get 1 red, 9 yellow and 1 yellow and 9 reds and are able to formulate conjectures about what they noticed. Also, students begin to look for strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving all tasks, Grade 1 students accurately check for the reasonableness of their solutions during, and after completing the task.

Major **Supporting** **Additional** **Depth Opportunities**

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. **Make sense of problems and persevere in solving them.** • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; "Does this make sense?"	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
2. **Reason abstractly and quantitatively.** • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is ______related to ______? • What is the relationship between _______and ______? • What does _______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
3. **Construct viable arguments and critique the reasoning of others.** • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? How can we be sure that _________? / How could you prove that._____? Will it still work if._____? • What were you considering when._______? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about.______? • How could you demonstrate a counter-example?
4. **Model with mathematics.** • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

Major **Supporting** **Additional** **Depth Opportunities**

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. **Use appropriate tools strategically.** • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative? • Why was it helpful to use.______? • What can using a _____ show us, that ____may not? • In what situations might it be more informative or helpful to use.______?
6. **Attend to precision.** • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Is there a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties can you use to explain.____? • How could you test your solution to see if it answers the problem?
7. **Look for and make use of structure.** • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	• What observations do you make about._____? • What do you notice when.______? • What parts of the problem might you eliminate, simplify? • What patterns do you find in.______? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to.________? • In what ways does this problem connect to other mathematical concepts?
8. **Look for and express regularity in repeated reasoning.** • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Will the same strategy work in other situations? • Is this always true, sometimes true or never true? • How would we prove that._________? • What do you notice about._________? • What is happening in this situation? • What would happen if.________? • What Is there a mathematical rule for.______? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Critical Areas for Mathematics in 1st Grade

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
(1.OA.1, 1.OA.2, 1.OA.3, 1.OA.4, 1.OA.5, 1.OA.6, 1.OA.7, 1.OA.8)

2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
(1.NBT.1, 1.NBT.2, 1.NBT.3, 1.NBT.4, 1.NBT.5, 1.NBT.6)

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement. (Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.
(1.MD.1, 1.MD.2)

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website

Grade 1 Content Standards Overview

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving addition and subtraction.
  OA.1 OA.2
- Understand & apply properties of operations and the relationship between addition & subtraction.
  OA.3 OA.4
- Add and subtract within 20.
  OA.5 OA.6
- Work with addition and subtraction equations.
  OA.7 OA.8

Number and Operations in Base Ten (NBT)

- Extend the counting sequence.
  NBT.T
- Understand the place value system.
  NBT.2 NBT.3
- Use place value understanding and properties of operations to add and subtract.
  NBT.4 NBT.5 NBT.6

Measurement and Data (MD)

- Measure lengths indirectly and by iterating length units.
  MD.1 MD.2
- Tell and write time.
  MD.3
- Represent and interpret data.
  MD.4

Geometry (GE)

- Reason with shapes and their attributes.
  G.1 G.2 G.3

See: Illustrative Mathematics for sample tasks.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Represent and solve problems involving addition and subtraction.*

Standard: Grade 1. OA.1

Use addition and subtraction within 20 to solve word problems involving situations of **adding to**, **taking from**, **putting together**, **taking apart**, and **comparing**, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to the First Grade Area of Focus #1, Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20.
- This cluster is connected to Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from in Kindergarten, to Work with addition and subtraction equations in Grade 1, and to Represent and solve problems involving addition and subtraction and Add and subtract within 20 in Grade 2.

Explanation and Examples:

This standard builds on the work in Kindergarten by having students use a variety of mathematical representations (e.g., objects, drawings, and equations) during their work. The unknown symbols should include boxes or pictures, and not letters.

Teachers should be aware of the three types of problems, Appendix Table 1 and provide multiple experiences for their students solving ALL three types of problems. The three types of addition and subtraction problems: **Result Unknown, Change Unknown, and Start Unknown**.

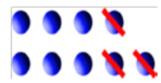
Use informal language (and, minus/subtract, the same as) to describe joining situations (putting together) and separating situations (breaking apart).

Use the addition symbol (+) to represent joining situations, the subtraction symbol (-) to represent separating situations, and the equal sign (=) to represent a relationship regarding quantity between one side of the equation and the other.

A helpful strategy is for students to recognize sets of objects in common patterned arrangements (0-10) to tell how many without counting (subitizing).

Contextual problems that are closely connected to students' lives should be used to develop fluency with addition and subtraction. Table 1 page 46, describes the four different addition and subtraction situations and their relationship to the position of the unknown. Students use objects or drawings to represent the different situations.

- **<u>Take From example</u>**: Avery has 9 balls. She gave 3 to Susan. How many balls does Avery have now?



- **<u>Compare example</u>**: Avery has 9 balls. Susan has 3 balls. How many more balls does Avery have than Susan? A student will use 9 objects to represent Avery's 9 balls and 3 objects to represent Susan's 3 balls. Then they will compare the 2 sets of objects.

Note that even though the modeling of the two problems above is different, the equation,
9 - 3 = ?, can represent both situations yet the compare example can also be represented by 3 + ? = 9 (*How many more do I need to make 9*?)
It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.

- Result Unknown, Total Unknown, and Both Addends Unknown problems are the least complex for students.
- The next level of difficulty includes Change Unknown, Addend Unknown, and Difference Unknown
- The most difficult are Start Unknown and versions of Bigger and Smaller Unknown (compare problems).

More Examples:

Result Unknown	**Change Unknown**	**Start Unknown**
There are 9 students on the playground. Then 8 more students showed up. How many students are there now? $9 + 8 = ?$	There are 9 students on the playground. Some more students showed up. There are now 17 students. How many students came? $9 + ? = 17$	Here are some students on the playground. Then 8 more students came. There are now 17 students. How many students were on the playground at the beginning? $? + 8 = 17$

Please see Table 1, appendix for additional examples. The level of difficulty for these problems can be differentiated by using smaller numbers (up to 10) or larger numbers (up to 20).

Instructional Strategies: **(1.OA. 1 & 2)**

Learning to ***mathematize*** (*the process of seeing and focusing on the mathematical aspects and ignoring the nonmathematical aspects. Mathematizing in first grade: Solving problems, reasoning, and Communicating, Connecting, and Representing Ideas.*

Modeling addition and subtraction situations with objects, fingers, and drawings is the foundation for algebraic problem solving. More difficult types of problems situations (change and collection situations) should be given from grade 1 on.

Provide opportunities for students to participate in shared problem-solving activities to solve word problems. Collaborate in small groups to develop problem-solving strategies using a variety of models such as drawings, words, and equations with symbols for the unknown numbers to find the solutions. Additionally students need the opportunity to explain, write and reflect on their problem-solving strategies.

The situations for the addition and subtraction story problems should involve sums and differences less than or equal to 20 using the numbers 0 to 20. They need to align with the 12 situations found in Table 1 page 46 of this document (Also in *Common Core State Standards (CCSS) for Mathematics, page 88)*.

Students need the opportunity of writing and solving story problems involving three addends with a sum that is less than or equal to 20. For example, each student writes or draws a problem in which three whole things are being combined. The students exchange their problems with other students, solving them individually and then discussing their models and solution strategies. Now both students work together to solve each problem using a different strategy.

It is important to emphasize the most critical problem-solving strategy—**understand the situation** and represent the problem (e.g. use counters, cubes, or drawings). Key-word strategies in which children focus only on one or a few words will not work with algebraic problems. Teachers need to help all children move beyond such limiting strategies by emphasizing **understanding & representing** the situation.

Literature is an excellent way to incorporate problem-solving in a context that young students can understand. Many literature books that include mathematical ideas and concepts have been written in recent years.

For Grade 1, the incorporation of books that contain a problem situation involving addition and subtraction with numbers 0 to 20 should be included in the curriculum. Use the situations found in Table 1, in the Appendix for guidance in selecting appropriate books. As the teacher reads the story, students use a variety of manipulatives, drawings, or equations to model and find the solution to problems from the story.

Expect and support children's ability to make meaning and mathematize the real world and create a nurturing and helpful math-talk community.

Tools/Resources

For detailed information see Operations & Algebraic Thinking Learning Progression:
http://commoncoretools.me/wp-content/uploads/2011/04/ccss_progression_nbt_2011_04_073_corrected2.pdf

1.OA.A.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

1.OA At the Park
1.OA Boys and Girls, Variation 1
1.OA Maria's Marbles
1.OA Sharing Markers
1.OA Finding a Chair
1.OA Boys and Girls, Variation 2
1.OA The Pet Snake
1.OA Measuring Blocks
1.MD Growing Bean Plants
1.OA 20 Tickets
1.OA Field Day Scarcity
1.OA School Supplies
1.OA Link-Cube Addition

[1.OA Measuring Blocks](#)
[1.OA Peyton's Books](#)

Common Misconceptions:

Many children misunderstand the meaning of the equal sign. The equal sign means “is the same as” or the left side of the equation *balances* or is the *same as* right side of the equal sign. However, most primary students believe the equal sign tells you that the “answer is coming up” to the right of the equal sign. This misconception is over-generalized by only seeing examples of number sentences with an operation to the left of the equal sign and the answer on the right.

First graders need to see equations written multiple ways, for example $5 + 7 = 12$ and $12 = 5 + 7$.

A second misconception that many students have is that it is valid to assume that a key word or phrase in a problem suggests the same operation will be used every time. For example, they might assume that the word *left* always means that subtraction must be used to find a solution. Providing problems in which key words like this are used to represent different operations is essential. For example, the use of the word *left* in this problem does not indicate subtraction as a solution method: Jose took the 8 stickers he no longer wanted and gave them to Anna. Now Jose has 11 stickers *left*. How many stickers did Jose have to begin with?

Students need to analyze word problems and make sense of them, rather than look for “tricks” to help them decide which operation to use. Avoid teaching key words to solve problems, instead emphasize understanding the situation.

Domain: Operations and Algebraic Thinking (OA)

***Cluster:** Represent and solve problems involving addition and subtraction.*

Standard: Grade 1.OA.2

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1. OA.1

Explanation and Examples:

This standard asks students to add (join) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations. This objective does address multi-step word problems.

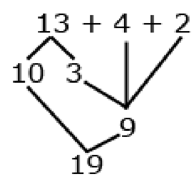
Example:

There are cookies on the plate. There are 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies are there total?

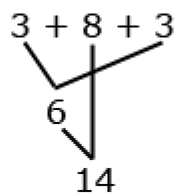
Student 1 **Adding with a Ten Frame and Counters**	**Student 2** **Look for ways to make 10**	**Student 3** **Number Path (See *Number Path* explanation in the following section)**
I put 4 counters on the 10 Frame for the oatmeal raisin cookies. Then, I put 5 different color counters on the 10-Frame for the chocolate chip cookies. Then, I put another 6 color counters out for the gingerbread cookies. Only one of the gingerbread cookies fit, so I had 5 left over. One 10-Frame and five leftover makes 15 cookies. (Students use concrete models).	I know that 4 and 6 equal 10, so the oatmeal raisin and gingerbread equals 10 cookies. Then, I add the 5 chocolate chip cookies and get 15 total cookies.	I counted on the number path. First, I counted 4, and then I counted 5 more and landed on 9. Then, I counted 6 more and landed on 15. So there were 15 total cookies.

To further students' understanding of the concept of addition, students create word problems with three addends. They can also increase their estimation skills by creating problems in which the sum is less than 5, 10 or 20. They use properties of operations and different strategies to find the sum of three whole numbers such as:

- Counting on and counting on again (e.g., to add $3 + 2 + 4$ a student writes $3 + 2 + 4 =?$ and thinks, "3, 4, 5, that's 2 more, 6, 7, 8, 9 that's 4 more so $3 + 2 + 4 = 9$."
- Making tens (e.g., $4 + 8 + 6 = 4 + 6 + 8 = 10 + 8 = 18$)
- Using "plus 10, minus 1" to add 9 (e.g., $3 + 9 + 6$ A student thinks, "9 is close to 10 so I am going to add 10 plus 3 plus 6 which gives me 19. Since I added 1 too many, I need to take 1 away so the answer is 18).
- Decomposing numbers between 10 and 20 into 1 ten plus some ones to facilitate adding the ones



- Using doubles



Students will use different strategies to add the 6 and 8.

- Using near doubles (e.g.,$5 + 6 + 3 = 5 + 5 + 1 + 3 = 10 + 4 = 14$)

Students may use document cameras to display their combining strategies. This gives them the opportunity to communicate and justify their thinking.

Instructional Strategies:
Children need many opportunities to use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number paths, to model "part-whole", "adding to," "taking away from", and "comparing situations to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children need to understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as "counting on" two).

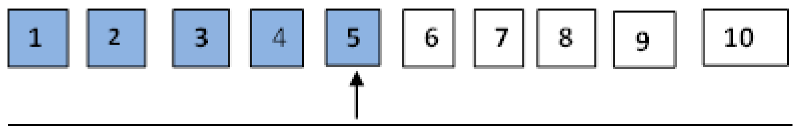
"Number paths" were used in the examples rather than "number lines". A great deal of confusion arises about what the term *number line* means. It is recommended that number lines not be used until grade 2 because they are conceptually too difficult for younger children.

In early childhood materials including Kindergarten, the term *number line or mental number line* often really means a **number path**, such as in the common early childhood games where numbers are put on squares and children move along such a numbered path. Such number paths are counting models in which things are counted. Each square is a thing that can be counted, so these are appropriate for children age two through grade 1.

A number path and a number line are shown below along with the meanings that children must understand and relate when using these models. A number line is a **length model** such as a ruler or a bar graph in which numbers are represented by the length from zero along a line segmented into equal lengths. Children need to count the **length units** on a number line, not the numbers.

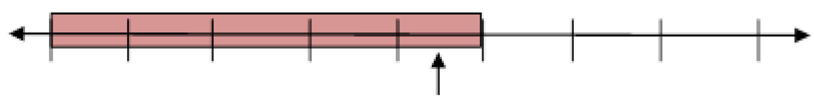
Young children have difficulties with such a number line representation because they have difficulty seeing the units—they need to see things, so they focus on the numbers or the segmenting marks instead of on the lengths. Thus, they may count the starting point 0 and then be off by one, or they may focus on the spaces and be confused by the location of the numbers at the ends of the spaces.

A Number Path: Counting Things



Count and Cardinal Word Meanings When Counting Things in a Number Path
Count word reference: *"The arrow points to the square where I say five."*
Cardinal word reference: *"These are five squares."* (shaded squares)

A Number Line: Counting Length Units



Count and Measure Word Meanings When Counting Unit Lengths on a Number Line
Count word reference: *"The arrow points to the **unit length** where I say five*".
Measure word reference: "These are five ***unit lengths***." (shaded lengths)

Consider also teaching *subtraction as an unknown addend*: students can always solve subtraction problems by a forward method that finds the unknown. The counting down or back is difficult and error-prone. Help children learn and use the more accurate forward methods

Resources/Tools

1.OA.A.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA Daisies in vases
1.OA, NBT The Very Hungry Caterpillar
"Creating Story Problems", Georgia Department of Education.

Student will apply comprehensions skills to story problems and the understanding of addition and subtraction situations and operations to solve and to write problems that include: part-part- whole, comparing, grouping, doubling, counting on and counting back situations
Focus in Grade 1, NCTM

Common Misconceptions:

As mentioned earlier, many children misunderstand the meaning of the equal sign. Emphasize the conceptual understanding of the equal sign, e.g., "is the same as". Most primary students believe the equal sign tells them that the “answer is coming up” on the right side of the equal sign. They also believe the equal sign can only be on the right side of an equation. This misconception is over-generalized by only seeing examples of number sentences with an operation to the left of the equal sign and the answer on the right.

It is important that first graders see equations written multiple ways.

Example: $5 + 7 = 12$ and $12 = 5 + 7$ and $3 + 2 + 7 = 7 + 5$

A second misconception that many students have is that it is valid to assume that a key word or phrase in a problem suggests the same operation will be used every time. For example, they might assume that the word *left* always means that subtraction must be used to find a solution. Providing problems in which key words like this are used to represent different operations is essential. For example, the use of the word *left* in this problem does not indicate subtraction as a solution method:

Seth took the 8 stickers he no longer wanted and gave them to Anna. Now Seth has 11 stickers *left*. How many stickers did Seth have to begin with? Students need to analyze word problems and avoid using key words to solve them.

See **Number Line and Number Path** explanations and examples above for misconceptions on use of the Number Line (linear/length model) for counting, adding, etc.

Domain: Operations and Algebraic Thinking (OA)

***Cluster**: Understand and apply properties of operations and the relationship between addition and subtraction.*

Standard: Grade 1.OA.3

Apply properties of operations as strategies to add and subtract. *Examples: If* $8 + 3 = 11$ *is known, then* $3 + 8 = 11$ *is also known. (Commutative property of addition.) To add,* $2 + 6 + 4$*the second two numbers can be added to make a ten, so* $2 + 6 + 4 = 2 + 10 = 12$*. (Associative property of addition.) (*Students need not use formal terms for these properties.)

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #1**, Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20**.
- This cluster is connected to *Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from* in Kindergarten, to *Add and subtract within 20* and *Use place value understanding and properties of operations to add and subtract* in Grade 1 and to *Use place value understanding and properties of operations to add and subtract* in Grade 2.

Explanation and Examples:

This standard asks students to apply properties of operations as strategies to **add** and **subtract**. Students do not need to use formal terms for these properties. Students should use mathematical tools, such as cubes and counters, and representations such as the number path and a 100 chart to model these ideas.

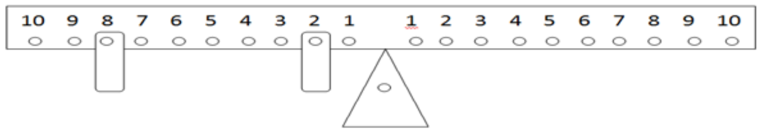
Students use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Example:

Student can build a tower of 8 green cubes and 3 yellow cubes and another tower of 3 yellow and 8 green cubes to show that order does not change the result in the operation of addition. Students can also use cubes of 3 different colors to prove that $(2 + 6) + 4$ is equivalent to $2 + (6 + 4)$ and then to prove $2 + 6 + 4 = 2 + 10$. Students should understand the important ideas of the following properties:

- Identity property of addition (e.g., $6 = 6 + 0$)
- Identity property of subtraction (e.g., $9 - 0 = 9$)
- Commutative property of addition--Order does not matter when you add numbers. (e.g. $4 + 5 = 5 + 4$)
- Associative property of addition--When adding a string of numbers you can add any two numbers first. (e.g., $3 + 9 + 1 = 3 + 10 = 13$)

Student 1
Using a number balance to investigate the commutative property. If I put a weight on 8 *first* and *then* 2, I think that it will balance if I put a weight on 2 *first* this time *then* on 8.



Students need several experiences investigating whether the commutative property works with subtraction. The intent is not for students to experiment with negative numbers but only to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should recognize that they will be working with numbers later on that will allow them to subtract larger numbers from smaller numbers. However, in first grade students do not work with negative numbers.

Instructional Strategies: 1.OA. 3-4

Instruction needs to focus on lessons that help students to discover and apply the commutative and associative properties as strategies for solving addition problems. It is not necessary for students to learn the names for these properties. It is important for students to represent, share, discuss, and compare their strategies as a class.

The second focus is using the relationship between addition and subtraction as a strategy to solve unknown-addend problems. Students naturally connect counting on to solving subtraction problems. For the problem "$15 - 7 = ?$" they think about the number they have to add to 7 to get to 15. First graders should be working with sums and differences less than or equal to 20 using the numbers 0 to 20.

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The second focus is using the relationship between addition and subtraction as a strategy to solve unknown-addend problems. Students naturally connect counting on to solving subtraction problems. For the problem "$15 - 7 = ?$" they think about the number they have to add to 7 to get to 15. First graders should be working with sums and differences less than or equal to 20 using the numbers 0 to 20.

Provide investigations that require students to identify and then apply a pattern or structure of mathematics.

- For example, pose a string of addition and subtraction problems involving the same three numbers chosen from the numbers 0 to 20, like $4 + 13 = 17$ and $13 + 4 = 17$.
- Students analyze number patterns and create conjectures or guesses. Have students choose other combinations of three numbers and explore to see if the patterns work for all numbers 0 to 20.
- Students then share and discuss their reasoning. Be sure to highlight students' uses of the commutative and associative properties and the relationship between addition and subtraction.

Expand the student work to three or more addends to provide the opportunities to change the order and/or groupings to make tens. This will allow the connections between place-value models and the properties of operations for addition to be seen. Understanding the commutative and associative properties builds flexibility for computation and estimation, a key element of number sense.

Provide multiple opportunities for students to study the relationship between addition and subtraction in a variety of ways, including games, modeling and real-world situations. Students need to understand that addition and subtraction are related, and that subtraction can be used to solve problems where the addend is unknown.

Tools/Resources

1.OA.B. Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA Fact Families

1.OA Fact Families with Pictures

1.OA.B.3. Apply properties of operations as strategies to add and subtract.* Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)

1.OA Domino Addition

Common Misconceptions:

A common misconception is that the commutative property applies to subtraction. After students have discovered and applied the commutative property for addition, ask them to investigate whether this property works for subtraction. Have students share and discuss their reasoning and guide them to conclude that the commutative property does not apply to subtraction.

First graders might have informally encountered negative numbers in their lives, so they think they can take away more than the number of items in a given set, resulting in a negative number(below zero). Provide many problems situations where students take away all objects from a set, e.g. 19 - 19 = 0 and focus on the meaning of 0 objects and 0 as a number. Ask students to discuss whether they can take away more objects than what they have.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Understand and apply properties of operations and the relationship between addition and subtraction.*

Standard: Grade 1.OA.4

Understand subtraction as an unknown-addend problem.
For example, subtract $10 - 8$ *by finding the number that makes 10 when added to 8. Add and subtract within 20.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2   Reason abstractly and quantitatively.
- ✓ MP.7   Look for and make use of structure.
- ✓ MP.8   Look for and express regularity in repeated reasoning.

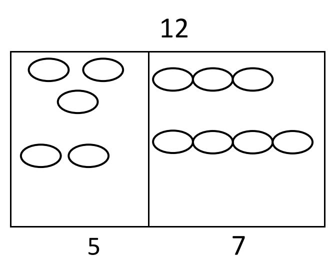
Connections: See Grade 1.OA.3

Explanation and Examples:

This standard asks for students to use subtraction in the context of unknown addend problems. When determining the answer to a subtraction problem, $12 - 5$, students think, "If I have 5, how many more do I need to make 12?" Encouraging students to record this symbolically, $5 + ? = 12$, will develop their understanding of the relationship between addition and subtraction.

Some strategies they may use are counting objects, creating drawings, counting up, using number path or 10 frames to determine an answer. Refer to Table 1, page 46 to consider the level of difficulty of this standard.

Example:
$12 - 5 = ?$ could be expressed as $5 + ? = 12$. Students should use cubes and counters, and representations such as the number path and the100 chart, to model and solve problems involving the inverse relationship between addition and subtraction.

Student 1	Student 2	Student 3
I used a ten frame. I started with 5 counters. I now that I had to have 12, which is one full ten frame and two left over. I needed 7 counters, so $12 - 5 = 7$	I used a part-part-whole diagram. I put 5 counters on one side. I wrote 12 above the diagram. I put counters into the other side until there were 12 in all. I know I put 7 counters into the other side, so $12 - 5 = 7$. 	Draw a number path. I started at 5 and counted until I reached 12. I counted 7 numbers, so I knew that $12 - 5 = 7$.

Instructional Strategies: **See Grade 1.OA.3**

Common Misconceptions: See Grade 1.OA.3

Domain: Operations and Algebraic Thinking (OA)

***Cluster**: Add and subtract within 20.*

Standard: Grade 1.OA.5

Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #1, **Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20**.
- This cluster is connected to all clusters in the Counting and Cardinality Domain, *Understand addition as putting together and adding to, and understanding subtraction as taking apart and taking from* and *work with numbers 11-19 to gain foundations for place value* in Kindergarten, to *Understand and apply properties of operations and the relationship between addition and subtraction in* Grade 1 and to *Add and subtract within 20* and *Use place value understanding and properties of operations to add and subtract* in Grade 2.

Explanation and Examples:

This standard asks for students to make a connection between counting and adding and subtraction. Students use various counting strategies, including **counting all and counting on** with numbers up to 20. This standard calls for students to move beyond counting all and become comfortable at counting on. The counting all strategy requires students to count an entire set. The counting and counting back strategies occur when students are able to hold the "start number" in their head and count on from that number.

Students' multiple experiences with counting may hinder their understanding of **counting on** as connected to addition and subtraction. To help them make these connections when students count on 3 from 4, they should write this as $4 + 3 = 7$. When students count on for subtraction (3) from 7, they should connect this to $7 - 3 = 4$. Students write $7 - 3 = ?$ and think I *count on* $3 + ? = 7$.

Additional Example: $\mathbf{5 + 3 = ?}$

Student 1	Student 2
Counts all $5 = 2 + ?$. The student counts five counters. The student adds two more. The student counts 1,2,3,4,5,6,7 to get the answer.	Counts On $5 + 2 = ?$. Student count five counters. The student adds the first counters & says 6, then adds another counter & says 7. The student knows the answer is 7 since they counted on 2.

See Levels of Children's Addition and Subtraction Methods (NCTM Focus in Grade 1)

Levels of Children's' Addition and Subtraction Methods

	8 + 6 = 14	**14 – 8 = 6**
Level 1: **Count all**	1 2 3 4 5 6 7 8 1 2 3 4 5 6 1 2 3 4 5 6 7 8 9 10 11 12 13 14	1 2 3 4 5 6 7 8 9 10 11 12 13 14 1 2 3 4 5 6 6 7 8 1 2 3 4 5 6
Level 2: **Count on**	8 8 9 10 11 12 13 14	To solve $14 - 8$: I count on $8 + ? = 14$ I took away 8. 8 to 14 is 6, so $14 - 8 + 6$.
Level 3: Recompose	1	$14 - 8$: I make a ten for $8 + ? = 14$ $8 + 2 + 4$ 6 $8 + ? = 14$
Doubles ± *n*	$6 + 8 = ?$ $6 + 6 + 2 = ?$ $12 + 2 + 14$	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful when counting; it makes subtraction as easy as addition. The use of "touch points or touch math" should be avoided since it encourages students to stay at Level 1.

Instructional Strategies: (Grade 1.OA.5-6)

Provide many experiences for students to construct strategies to solve the different problem types illustrated in Table 1 in the Appendix. These experiences should help students combine their procedural and conceptual understandings.

Have students invent and refine their strategies for solving problems involving sums and differences less than or equal to 20 using the numbers 0 to 20. Ask them to explain and compare their strategies as a class.

Provide multiple and varied experiences that will help students develop a strong sense of numbers based on comprehension – **not rules and procedures**. Number sense is a blend of comprehension of numbers and operations and fluency with numbers and operations. Students gain computational fluency (using efficient and accurate methods for computing) as they come to understand the role and meaning of arithmetic operations in number systems.

Primary students should come to understand addition and subtraction as they connect counting and number sequence to these operations. Addition and subtraction also involve part to whole relationships. Students' understanding that the whole is made up of parts is connected to decomposing and composing numbers.

Provide numerous opportunities for students to use the *counting on* strategy for solving addition and subtraction problems. For example, provide a ten frame showing 5 colored dots in one row. Students add 3 dots of a different color to the next row and write $5 + 3$. Ask students to count on from 5 to find the total number of dots. Then have them add an equal sign and the number eight to $5 + 3$ to form the equation $5 + 3 = 8$.

Ask students to verbally explain how counting on helps to add one part to another part to find a sum. Discourage students from inventing a counting back strategy for subtraction because it is difficult and leads to errors.

Instructional Resources/Tools

1.OA.C.5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

1.OA, NBT The Very Hungry Caterpillar

Five-frame and Ten-frame

A variety of objects for counting

A variety of objects for modeling and solving addition and subtraction problems

Table 1, Appendix

Common Misconceptions

Students ignore the need for regrouping when subtracting with numbers 0 to 20 and think that they should always subtract a smaller number from a larger number. For example, students solve 15 – 7 by subtracting 5 from 7 and 0 (0 tens) from 1 to get 12 as the incorrect answer. Students need to relate their understanding of place-value concepts and grouping in tens and ones to their steps for subtraction. They need to show these relationships for each step using mathematical drawings, ten-frames or base-ten blocks so they can understand an efficient strategy for multi-digit subtraction.

Domain: Operations and Algebraic Thinking (OA)

***Cluster**: Add and subtract within 20.*

Standard: Grade 1.OA.6

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as **counting on**; **making ten** (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

This is a required fluency for Grade 1.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1. OA.6

Explanation and Examples:

This standard is strongly connected to all the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 10 and having experiences adding and subtracting within 20.

By studying patterns and relationships in addition facts and relating addition and subtraction, students build a foundation for fluency with addition and subtraction facts. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly (use of different strategies), accurately, and efficiently.

The use of objects, diagrams, or interactive whiteboards and various strategies will help students develop fluency. It is important for students to be able to use a **variety** of strategies when adding and subtracting numbers within 20. Students should have ample experiences modeling these operations before working on fluency.

Algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem or when they use addition combinations they know to solve more difficult problems. Students begin to consider the relationship between the parts. For example, students notice that the whole remains the same, as one part increases the other part decreases. $5 + 2 = 4 + 3$

Instructional Strategies:

The use of objects, diagrams, or interactive whiteboards and various strategies will help students develop fluency. It is important for students to be able to use a **variety** of strategies when adding and subtracting numbers within 20. Students should have ample experiences modeling these operations before working on fluency.

Teacher could differentiate using smaller numbers. Also, it is important to move beyond the strategy of counting on, which is considered a less important skill than the ones here in 1.OA.6. Many times teachers think that counting on is all a child needs, when it is really not much better skill than counting all and can becomes a hindrance when working with larger numbers.

Example: $\mathbf{8 + 7} = ?$

Student 1	**Student 2**
Making 10 and Decomposing a Number I know that 8 plus 2 is 10 so I decomposed (broke) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10 and then added the 5 to get 15, $8 + 7 = (8 + 2) + 5 = 10 + 5 = 15$	Creating an Easier Problem with Known Sums I know 8+7 is 7+1. I also know that 7 and 7 equals 14 and then I added 1 more to get 15. $8 + 7 = (7 + 7) + 1 = 15$
Student 1	**Student 2**
Decomposing the Numbers when You Subtract I know that 14 minus 4 is 10 so I broke the 6 up into 4 and 2. 14 minus 4 is 10, Then I take away 2 more to get 8. $14 - 6 = (14 - 4) - 2 + 10 - 2 + 8$	Relationship Between Addition and Subtraction 6plus ___is 14. I know that 6 plus 8 is 14, so that means 14 minus 6 is 8. $6 + 8 = 14 \text{ so } 14 - 6 + 8$

*Algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem or when they use addition combinations they know to solve more difficult problems. Students begin to consider the relationship between the parts. For example, students notice that the whole remains the same, as one part increases the other part decreases.

$$5 + 2 = 4 + 3$$

Tools/Resources:

For detailed information, see Learning Progressions Operations and Algebraic Thinking:

1.OA $20 Dot Map

1.OA Making a ten

See also: "Ten is Our Friend", NSCM, Great Tasks for Mathematics K-5, (2013).

Common Misconceptions: See Grade 1. OA.5

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Work with addition and subtraction equations.*

Standard: Grade 1.OA.7

Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false?

$$6 = 6$$
$$7 = 8 - 1$$
$$5 + 2 = 2 + 5$$
$$4 + 1 = 5 + 2.$$

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #1**, Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20**.
- This cluster is connected to *Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from* in Kindergarten, to *Represent and solve problems involving addition and subtraction* in Grade 1, and to *Represent and solve problems involving addition and subtraction* and *Add and subtract within 20* in Grade 2.

Explanation and Examples:

This standard calls for students to work with the concept of equality by identifying whether **equations** are **true** or **false.** Therefore, students need to understand that the equal sign does not mean "answer comes next", but rather that the equal sign signifies a relationship between the left and right side of the equation. Interchanging the language of "equal to" and "the same as" as well as "not equal to" and "not the same as" will help students grasp the meaning of the equal sign. Students should understand that *"*equality*"* means "the same quantity as". In order for students to avoid the common pitfall that the equal sign means "to do something" or that the equal sign means "the answer is," they need to be able to:

- Express their understanding of the meaning of the equal sign
- Accept sentences other than a + b = c as true ($a = a, c = a + b, a = a + 0, a + b = b + a$)
- Know that the equal sign represents a relationship between two equal quantities
- Compare expressions without calculating

The number sentence $4 + 5 = 9$ can be read as, "Four plus five is the same amount as nine."
In addition, Students should be exposed to various representations of equations, such as:

- an operation on the left side of the equal sign and the answer on the right side
  ($5 + 8 = 13$) "Compose"
- an operation on the right side of the equal sign and the answer on the left side ($13 = 5 + 8$) "Decompose"
- numbers on both sides of the equal sign ($6 = 6$)
- operations on both sides of the equal sign ($5 + 2 = 4 + 3$).

Students need many opportunities to model equations using cubes, counters, drawings, etc.

These key skills are hierarchical in nature and need to be developed over time.

Experiences determining if equations are true or false help student develop these skills. Initially, students develop an understanding of the meaning of equality using models. However, the goal is for students to reason at a more abstract level. At all times students should justify their answers, make conjectures (e.g., if you add a number and then subtract that same number, you always get zero), and make estimations.

Once students have a solid foundation of the key skills listed above, they can begin to rewrite true/false statements using the symbols, < and >.

Examples of true and false statements:

- $7 = 8 - 1$
- $8 = 8$
- $1 + 1 + 3 = 7$
- $4 + 3 = 3 + 4$
- $6 - 1 = 1 - 6$
- $12 + 2 - 2 = 12$
- $9 + 3 = 10$
- $5 + 3 = 10 - 2$
- $3 + 4 + 5 = 3 + 5 + 4$
- $3 + 4 + 5 = 7 + 5$
- $13 = 10 + 4$
- $19 + 9 + 1 = 19$

Instructional Strategies:

Provide opportunities for students use objects of equal weight and a number balance to model equations for sums and differences less than or equal to 20 using the numbers 0 to 20. Give students equations in a variety of forms that are true and false. Include equations that show the identity property, commutative property of addition, and associative property of addition.

Students need not use formal terms for these properties.

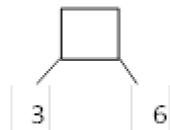
- $13 = 13$ Identity Property
- $8 + 6 = 6 + 8$ Commutative Property for Addition
- $3 + 7 + 4 = 10 + 4$ Associative Property for Addition

When asking students to determine whether the equations are true or false have them record their work with drawings. Students then compare their answers as a class and discuss their reasoning. Present equations recorded in a nontraditional way, like $13 = 16 - 3$ and $9 + 4 = 18 - 5$, then ask, “Is this true?.” Have students decide if the equation is true or false. Then as a class, students discuss their thinking that supports their answers.

Provide situations relevant to first graders for these problem types illustrated in Table 1 in the Appendix: Add to / Result Unknown, Take from / Start Unknown, and Add to / Result Unknown.

Demonstrate how students can use graphic organizers such as the *Math Mountain* (shown below) to help them think about problems. The *Math Mountain* shows a sum with diagonal lines going down to connect with the two addends, forming a triangular shape.

It shows two known quantities and one unknown quantity. Use various symbols, such as a square, to represent an unknown sum or addend in a horizontal equation. For example, here is a Take from / Start Unknown problem situation such as: Some markers were in a box. Matt took 3 markers to use. There are now 6 markers in the box. How many markers were in the box before? The teacher draws a square to represent the unknown sum and diagonal lines to the numbers 3 and 6.



Have students practice using the *Math Mountain* to organize their solutions to problems involving sums and differences less than or equal to 20 with the numbers 0 to 20. Then ask them to share their reactions to using the *Math Mountain.* Provide numerous experiences for students to compose and decompose numbers less than or equal to 20 using a variety of manipulatives. Have them represent their work with drawings, words, and numbers. Ask students to share their work and thinking with their classmates. Then ask the class to identify similarities and differences in the students' representations.

Tools/Resources:

1.OA Valid Equalities?
1.OA Using lengths to represent equality
1.OA Equality Number Sentences
1.OA, NBT The Very Hungry Caterpillar
1.OA 20 Tickets

Common Misconceptions:

Many students think that the equals sign means that an operation must be performed on the numbers on the left and the result of this operation is written on the right. They think that the equal sign is like an arrow that means *becomes* and one number cannot be alone on the left. Students often ignore the equal sign in equations that are written in a nontraditional way.

For instance, students find the incorrect value for the unknown in the equation $9 = \Delta - 5$ by thinking $9 - 4 = 4$. It is important to provide equations with a single number on the left as in $18 = 10 + 8$. Showing pairs of equations such as $11 = 7 + 4$ and $7 + 4 = 11$ gives students experiences with the meaning of the equal sign as *is the same as* and equations with one number to the left.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Work with addition and subtraction equations.*

Standard: Grade 1.OA.8

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations:*

$$8 + ? = 11, 5 = ? - 3, 6 + 6 = ?$$

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1. OA.7

Explanation and Examples:

This standard extends the work that students do in 1.OA.4 by relating addition and subtraction as related operations for situations with an unknown. This standard builds upon the "think addition" for subtraction problems as explained by **Student 2 in 1.OA.6.**

Student 1
$5 = ? - 3$
I know that 5 plus 3 is 8.
So, 8 minus 3 is 5.

Students need to understand the meaning of the equal sign and know that the quantity on one side of the equal sign must be the same quantity on the other side of the equal sign. They should be exposed to problems with the unknown in different positions. Having students create word problems for given equations will help them make sense of the equation and develop strategic thinking.

Examples of possible student "think-throughs":

- $8 + ? = 11$: "8 and some number is the same as 11. 8 and 2 is 10 and 1 more makes 11. So the answer is 3."
- $5 = ? - 3$: "This equation means I had some cookies and I ate 3 of them. Now I have 5. How many cookies did I have to start with? Since I have 5 left and I ate 3, I know I started with 8 because I count on from 5. . . 6, 7, 8."

All students need to show a representation of the problem and communicate and justify their thinking.

Tools/Resources

1.OA Find the Missing Number

1.OA Kiri's Mathematics Match Game

Instructional Strategies: See Grade1.OA.7

Common Misconceptions: See Grade 1.OA.7

Domain: Number and Operations in Base 10 (NBT)

***Cluster**: Extend the counting sequence.*

Standard: Grade 1.NBT.1

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #2**, Developing understanding of whole number relationships and place value, including grouping in tens and ones**.
- This cluster is connected to *Know number names and the count sequence* and *Compare numbers* in Kindergarten, and to *Understand place value* in Grade 2.

Explanation and Examples:

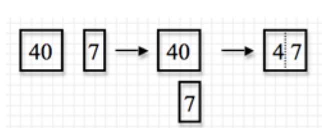
This standard calls for students to rote count forward to 120 by *Counting On* from any number less than 120. Students should have ample experiences with the hundreds chart to see patterns between numbers, such as all of the numbers in a column on the hundreds chart have the same digit in the ones place, and all of the numbers in a row have the same digit in the tens place.

This standard also calls for students to read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include cubes, place value (base 10) blocks, pictorial representations or other concrete materials. Students should use objects, words, and/or symbols to express their understanding of numbers.

As students are developing accurate counting strategies they are also building an understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). They extend their counting beyond 100 to count up to 120 by counting by 1s.

Some students may begin to count in groups of 10 (while other students may use groups of 2s or 5s to count). Counting in groups of 10 as well as grouping objects into 10 groups of 10 will develop students understanding of place value concepts.

After counting objects, students write the numeral or use numeral cards to represent the number. Given a numeral, students read the numeral, identify the quantity that each digit represents using numeral cards, and count out the given number of objects.



Students should experience counting from different starting points (e.g., start at 83; count to 120). To extend students' understanding of counting, they should be given opportunities to count backwards by ones and tens. They should also investigate patterns in the base 10 system.

Instructional Strategies:
In first grade, students build on their counting to 100 by ones and tens beginning with numbers other than 1 as they learned in Kindergarten. Students can start counting at any number less than 120 and continue to 120. It is important for students to connect different representations for the same quantity or number.

Students use materials to count by ones and tens to a build models that represent a number, then they connect this model to the number word and its representation as a written numeral. Students learn to use numerals to represent numbers by relating their place-value notation to their models.

They build on their experiences with numbers 0 to 20 in Kindergarten to create models for 21 to 120 with materials that can be grouped and material that are already grouped.

Students represent the quantities shown in the models by placing numerals in labeled hundreds, tens and ones columns. They eventually move to representing the numbers in standard form, where the group of hundreds, tens, then singles shown in the model matches the left-to-right order of digits in numbers.

Listen as students orally count to 120 and focus on their transitions between decades and the century number. These transitions will be signaled by a 9 and require new rules to be used to generate the next set of numbers. Students need to listen to their rhythm and pattern as they orally count so they can develop a strong number word list.

Extend hundreds charts by attaching a blank hundreds charts and writing the numbers 101 to 120 in the spaces following the same pattern as in the hundreds chart. Students can use these charts to connect the number symbols with their count words for numbers 1 to 120.

Post the number words in the classroom to help students read and write them.

Example 1: Place a handful of objects in front of your students and see how they count all. Do they have a system for keeping track of the items that have been counted? Are they demonstrating one-to-one correspondence?

Example 2: Tell the student that you have 18 objects under a cup. Hand them some more and have them count to see how many there are all together. Can the student start with 19 and count up? This is a stepping stone to adding, and understanding stories in context where one part is unknown. (Table 8 Appendix)

To develop concepts of number, have students estimate how many of something before counting. For example, before getting crayons have students estimate how many crayons are in the box. After estimating, partners should count to find a total. When students are first estimating, ask them if the amount is more than ten or less than ten, or more than 20 less than 20, etc. This helps them begin to get a sense of quantity.

Resources/Tools:

For detailed information, see Learning Progression Numbers and Operations in Base Ten:
1.NBT.A. Extend the counting sequence.
1.NBT Counting Circles II
1.NBT.A.1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
1.NBT "Crossing the Decade" Concentration
1.NBT Start/Stop Counting II
1.NBT Choral Counting II
1.NBT Hundred Chart Digit Game
1.NBT Number of the Day
1.NBT Where Do I Go?

Also see: ***Developing Counting***, Table 6 Appendix

Common Misconceptions:

Using the number 16 for example, children who do not have a well-established concept of place value, when asked to show with objects, will be able to show the 6 with six objects, when asked to show the one ten, will lay out only one additional object as a representation for the ten rather than 10 objects.

Some students might struggle with "seeing" the repeated pattern in our "system" as they count. AND, the names of our teen numbers do not follow the names of all other 2 digit numbers, since the names of the later decades, such as twenty, thirty, and fifth do not necessarily connect to two, three, and five. Recording the numbers can draw students' attention to the 0-9 pattern in each place, not just the ones.

To develop concepts of number, have students estimate how many of something before counting. For example, before getting crayons have students estimate how many crayons are in the box. After estimating, partners should count to find a total. When students are first estimating, ask them if the amount is more than ten or less than ten, or more than 20 less than 20, etc. This helps them begin to get a sense of quantity.

First graders may write "19" and mean "91". As they begin to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers.

To address the mistake of writing "19" for "9"1, teachers can demonstrate opportunities to "find mistakes", and question students. Example: "I'm reading your number and it says nineteen. Did you mean nineteen or ninety-one? How can you change the number so that it reads ninety-one?" This type of question allows students to analyze their mistakes and become precise (MP #6) as they write numbers to 120.

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand place value.*

Standard: Grade 1.NBT.2 a-c

Understand that the two digits of a two-digit number represent amounts of **tens** and **ones**. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

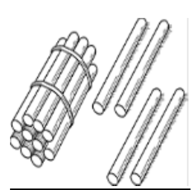
- This cluster is connected to the First Grade Critical Area of Focus #2, **Developing understanding of whole number relationships and place value, including grouping in tens and ones**.
- This cluster is connected to *Work with numbers 11-19 to gain foundations for place value* in Kindergarten, and to *Understand place value* in Grade 2.

Explanation and Examples:

Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones).

Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

1.NBT.2a asks students to unitize a group of ten ones as a whole unit: a ten. This is the foundation of the place value system. So, rather than seeing a group of ten cubes as ten individual cubes, the student is now asked to see those ten cubes as a bundle- one bundle of ten.



1.NBT.2b asks students to extend their work from Kindergarten when they composed and decomposed numbers from 11 to 19 into ten ones and some further ones. In Kindergarten, everything was thought of as individual units: "ones".

In First Grade, students are asked to unitize those ten individual ones as a whole unit: "*one* ten". Students in first grade explore the idea that the teen numbers (11 to 19) can be expressed as *one* ten and some leftover ones. Ample experiences with ten frames will help develop this concept.

Example:

For the number 12, do you have enough to make a ten? Would you have any leftover? If so, how many leftovers would you have?

Student 1	Student 2
I filled a ten frame to make one ten and had two counters left over. I had enough to make a ten with some leftover. The number 12 has 1 ten and 2 ones.	I counted out 12 place value cubes. I had enough to trade 10 cubes for a ten-rod (stick). I now have 1 ten-rod and 2 cubes left over. So the number 12 has 1 ten and 2 ones.

1.NBT.2c builds on the work of **1.NBT.2b.** Students should explore the idea that decade numbers (e.g. 10, 20, 30, 40) are groups of tens with no left over ones. Students can represent this with cubes or place value (base 10) rods.

(Most first grade students view the ten stick (numeration rod) as ONE. It is recommended to make a ten with unfix cubes or other materials that students can group. Provide students with opportunities to count books, cubes, pennies, etc. Counting 30 or more objects supports grouping to keep track of the number of objects.)

Understanding the concept of 10 is fundamental to children's mathematical development. Students need multiple opportunities counting 10 objects and "bundling" them into one group of ten. They count between 10 and 20 objects and make a bundle of 10 with or without some left over (this will help students who find it difficult to write teen numbers). Finally, students count any number of objects up to 99, making bundles of 10s with or without leftovers.

As students are representing the various amounts, it is important that an emphasis is placed on the language associated with the quantity. For example, 53 should be expressed in multiple ways such as 53 ones or 5 groups of ten with 3 ones leftover.

When students read numbers, they read them in standard form as well as using place value concepts. For example, 53 should be read as "fifty-three" as well as five tens, 3 ones. Reading 10, 20, 30, 40, 50 as "one ten, 2 tens, 3 tens, etc." helps students see the patterns in the number system.

Instructional Strategies:

Essential skills for students to develop include making tens (composing) and breaking a number into tens and ones (decomposing). Composing numbers by tens is foundational for representing numbers with numerals by writing the number of tens and the number of leftover ones.

Decomposing numbers by tens builds number sense and the awareness that the order of the digits is important. Composing and decomposing numbers involves number relationships and promotes flexibility with mental computation.

The beginning concepts of place value are developed in Grade 1 with the understanding of ones and tens. The major concept is that putting ten ones together makes a ten and that there is a way to write that down so the same number is always understood. Students move from counting by ones, to creating groups and ones, to tens and ones.

It is essential at this grade for students to see and use multiple representations of making tens using base-ten blocks, bundles of tens and ones, and ten-frames. Making the connections among the representations, the numerals and the words are very important. Students need to connect these different representations for the numbers 0 to 99.

- Groups of ones (single objects)
- Groups of 2 tens and 3 ones (2 ten-rods & 3 singles)
- Place Value Table,
- Write the Number,
- Read and Say the Number.

Students need to move through a progression of representations to learn a concept. They start with a **C**oncrete model, move to a pictorial or **R**epresentational model, then an **A**bstract model (**CRA).**

For example, ask students to place a handful of small objects in one region and a handful in another region. Next have them draw a picture of the objects in each region. They can draw a likeness of the objects or use a symbol for the objects in their drawing. Now have them count the physical objects or the objects in their drawings in each region and use numerals to represent the two counts. They also say and write the number word. Now students can compare the two numbers using an inequality symbol or an equal sign.

Tools /Resources

See: "Ten is Our Friend", NSCM, Great Tasks for Mathematics K-5, (2013).
For detailed information see NBT Learning Progressions
Also, see engageNY Modules

Common Misconceptions:

Often when students learn to use an aid (Pac Man, bird, alligator, etc.) for knowing which comparison sign (<, >, =) to use, the students don't associate the real meaning and name with the sign. The use of the learning aids must be accompanied by the connection to the names: < Less Than, > Greater Than, and = Equal To.

More importantly, students need to begin to develop the understanding of what it means for one number to be greater than another.

In Grade 1, it means that this number has more tens, or the same number of tens, but with more ones, making it greater. Additionally, the symbols are shortcuts for writing down this relationship. Finally, students need to begin to understand that both inequality symbols (<, >) can create true statements about any two numbers where one is greater/smaller than the other, ($15 < 28$ and $28 > 15$).

Domain: Number and Operations in Base Ten (NBT)

***Cluster:** Understand place value.*

Standard: Grade 1.NBT.3

Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1.NBT.2a-c

Explanation and Examples:

This standard builds on the work of **1.NBT.1** and **1.NBT.2** by having students compare two numbers by examining the amount of tens and ones in each number. Students are introduced to the symbols greater than (>), less than (<) and equal to (=). Students should have ample experiences communicating their comparisons using words, models and in context before using only symbols in this standard.

Example: 42__45

Student 1	Student 2
42 has 4 tens and 2 ones. 45 has 4 tens and 5 ones. They have the same number of tens, but 45 has more ones than 42. So 45 is greater than 42. So, $42 < 45$	42 is less than 45. I know this because when I count up I say 42 before I say 45. So, $42 < 45$

Students use concrete models that represent two sets of numbers. To compare, students first attend to the number of tens, then, if necessary, to the number of ones. Students may also use pictures, number paths, and spoken or written words to compare two numbers.

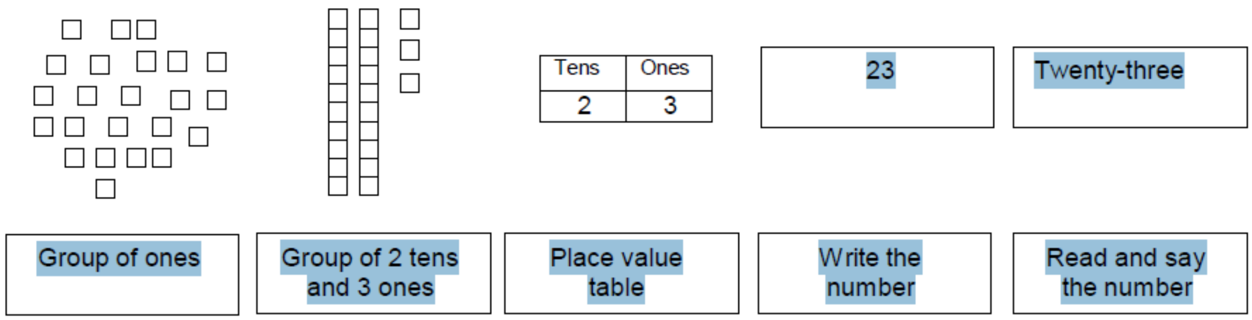
Comparative language includes but is not limited to more than, less than, greater than, most, greatest, least, same as, equal to and not equal to.

Instructional Strategies:

It is essential for students to develop the concepts of making tens (composing) and breaking a number into tens and ones (decomposing). Composing numbers by tens is foundational for representing numbers with numerals by writing the number of tens and the number of leftover ones. Decomposing numbers by tens builds number sense and the awareness that the order of the digits is important. Composing and decomposing numbers involves number relationships and promotes flexibility with mental computation.

Helping students put ten ones together makes a ten and that there is a way to write that down so the same number is always understood. Instruction needs to help students move from counting by ones, to creating groups and ones, to tens and ones. They need to see and use multiple representations of making tens using base-ten blocks, bundles of tens and ones, and ten-frames.

Making the connections among the representations, the numerals and the words are very important. Students need to connect these different representations for the numbers 0 to 99.
23 Twenty-three



Students need to move through a progression of representations to learn a concept. They start with a concrete model, move to a pictorial or representational model, then an abstract model.

Example

1. Ask students to place a handful of small objects in one region and a handful in another region. Next have them draw a picture of the objects in each region. They can draw a likeness of the objects or use a symbol for the objects in their drawing.
2. They count the physical objects or the objects in their drawings in each region and use numerals to represent the two counts.
3. They also say and write the number word.
4. Now students can compare the two numbers using an inequality symbol or an equal sign.

Tools/Resources:
See [engageNY Modules]

Common Misconceptions: See 1.NBT.2a-c

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding and properties of operations to add and subtract.*

Standard: Grade 1.NBT.4

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #1, **Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20**.
- This cluster connects to *Understand and apply properties of operations and the relationship between addition and subtraction* and *Understand place value* in Grade 1, and to *Add and subtract within 20*, *Use place value understanding and properties of operations to add and subtract* and *Relate addition and subtraction to length* in Grade 2.

Explanation and Examples:

This standard calls for students to use concrete models, drawings and place value strategies to add and subtract within 100. (Students should **not** be exposed to the standard algorithm of carrying or borrowing in first grade).

Students extend their number fact and place value strategies to add within 100. They represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. It is important for students to understand if they are adding a number that has 10s to a number with 10s, they will have more tens than they started with; the same applies to the ones.

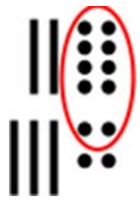
Also, students should be able to apply their place value skills to decompose numbers. For example, $17 + 12$ can be thought of 1 ten and 7 ones plus 1 ten and 2 ones. Numeral cards may help students decompose the numbers into 10s and 1s.
Students should be exposed to problems both in and out of context and presented in horizontal and vertical forms. As students are solving problems, it is important that they use language associated with proper place value (see example). They should always explain and justify their mathematical thinking both verbally and in a written format.

Estimating the solution prior to finding the answer focuses students on the meaning of the operation and helps them attend to the actual quantities. This standard focuses on developing addition - the intent is not to introduce traditional algorithms or rules.
Examples:

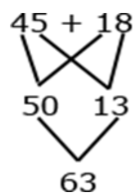
- $43 + 36$
  Student counts the 10s (10, 20, 30…70 or 1, 2, 3…7 tens) and then the 1s.

- 28
  <u>+34</u>

  Student thinks: 2 tens plus 3 tens is 5 tens or 50. S/he counts the ones and notices there is another 10 plus 2 more. 50 and 10 is 60 plus 2 more or 62.



- $45 + 18$
  Student thinks: Four 10s and one 10 are 5 tens or 50. Then 5 and 8 is $5 + 5 + 3(or\ 8 + 2 + 3)$ or 13. 50 and 13 is 6 tens plus 3 more or 63.



- 29
  <u>+14</u>

  Student thinks: "29 is almost 30. I added one to 29 to get to 30. 30 and 14 is 46. Since I added one to 29, I have to subtract one so the answer is 43."

- There are 37 children on the playground. 20more children show up. How many children are now on the playground? Student uses mental math. I started at 37 and counted on 3 to get to 40. Then, I added 20 which is 2 tens, to land on 60. So, there are 60 people on the playground.

- Same problem from above. I used a number path. I started on 37. Then I broke up 23 into 20 and 3 in my head. Next, I added 3 ones to get to 40. I then counted 10 to get to 50 and 10 more to get to 60. So, there are 60 children on the playground.

Instructional Strategies: **1.NBT.4-6**
It is important to provide multiple and varied experiences that will help students develop a strong sense of numbers based on **comprehension – not rules and procedures**.

Number sense is a blend of comprehension of numbers and operations and fluency with numbers and operations. Students gain computational fluency (using efficient and accurate methods for computing) when they are flexible and have many strategies from which to choose from, and as they come to understand the role and meaning of arithmetic operations in number systems.

Students should solve problems using **concrete** models and **drawings** to support and record their solutions. It is important for them to share the reasoning that supports their solution strategies with their classmates.

Students will usually move to using base-ten concepts, properties of operations, and the relationship between addition and subtraction to invent mental and written strategies for addition and subtraction. Help students share, explore, and record their invented strategies.

Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent. Encourage students to try the mental and written strategies created by their classmates. Students eventually need to choose efficient strategies to use to find accurate solutions.

Students should use and connect different representations when they solve a problem. They should start by building a concrete model to represent a problem. This will help them form a mental picture of the model. Now students move to using pictures and drawings to represent and solve the problem. If students skip the first step, building the concrete model, they might use finger counting to solve the problem. Finger counting is an inefficient strategy for adding within 100 and subtracting within multiples of 10 between10 and 90.

Have students connect a 0-99 chart or a 1-100 chart to their invented strategy for finding 10 more and 10 less than a given number. Ask them to record their strategy and explain their reasoning.

In a classroom environment where children know that they need to make sense of and explain their solution methods, children can invent methods for adding two-digit numbers with regrouping. Math drawings of tens and ones can serve as thinking tools in this process. But it is vital that children do not use math drawings or manipulatives like base-ten blocks or cubes organized into tens and ones in a rote manner just to get an answer. The explicit goal needs to be to develop a written method using numbers and to show the steps with numbers as well as with the quantities in the math drawings. The quantities of tens and ones need to be related to the written numerals in a solution method. Questions that can guide this process for children are these:

- *Will you get a new ten or not?*
- *Where will you write your new ten in your number problem?*

Tools/Resources :

Learning Progressions
See engageNY Modules

Common Misconceptions:

Students have alternate concepts of multi-digit numbers and see them as numbers independent of place value.

Example:

When counting or adding numbers, student read the number 32 as “thirty-two” and count out 32 objects to demonstrate the value of the number, but when asked to write the number in expanded form, they write “3+2”. Or when asked the value of the digits in the number they respond that the values are “3” and “2”.

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Use place value understanding and properties of operations to add and subtract*.

Standard: Grade 1.NBT.5

Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1.NBT.4

Explanation and Examples:

This standard builds on students' work with tens and ones and requires them to understand and apply the concept of 10 by mentally adding ten more and ten less than any number less than 100. This understanding leads to future place value concepts. It is critical for students to do this without counting.

Prior use of models such as base ten blocks, number path, and 100 charts helps facilitate understanding. Ample experiences with ten frames will also help students see the pattern involved when adding or subtracting 10 and USE these patterns to solve such problems.

Example:

There are 74 birds in the park. 10 birds fly away. How many are left?

Student 1: I used a 100s board. I started at 74. Then, because 10 birds flew away. I moved back one row. I landed on 64. So, there are 64 birds left in the park.

Student 2: I pictured 7 ten frames and 4 left over in my head. Since 10 birds flew away. I took one of the ten frames away. That left 6 ten frames and 4 left over. So, there are 64 birds left in the park.

Examples:

- 10 more than 43 is 53 because 53 is one more 10 than 43
- 10 less than 43 is 33 because 33 is one 10 less than 43

Tools/Resources:

See [engageNY Modules]

Instructional Strategies: **See Grade 1.NBT.4**

Common Misconceptions:
Students lack the concept that 10 in any position (place) makes one (group) and in the next position and vise-versa.

Example:
If students are asked to add a collection of 12 hundreds , 2 tens and 13 ones, students write 12213, possibly squeezing the 2 and the 13 together or separating the three numbers with some space.

171
+64
1135

Domain: Number and Operations in Base Ten (NBT)

***Cluster:* Use place value understanding and properties of operations to add and subtract.**

Standard: Grade 1.NBT.6

Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 1. NBT.4

Explanation and Examples:

This standard calls for students to use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g., 30, 40, 50).

This standard is foundation for future work in subtraction with greater numbers. Students should have multiple experiences representing numbers that are multiples of 10 (e.g. 90) with models or drawings. Then they subtract multiples of 10 (e.g. 20) using these representations or strategies based on place value. These opportunities develop fluency of addition and subtraction facts and reinforce counting up and back by 10s.

Examples:

- 70 - 30: Seven 10s take away three 10s is four 10s
- 80 - 50: 80, 70 (one 10), 60 (two 10s), 50 (three 10s), 40 (four 10s), 30 (five 10s)
- 60 - 40: I know that 4 + 2 is 6 so four 10s + two 10s is six 10s so 60 - 40 is 20

Example:

There are 60 students in the gym. 30 students leave. How many students are still in the gym?

Student 1	**Student 2**	**Student 3**	**Student 4**
I used a hundreds chart and started at 60. I moved up 3 rows to land on 30. There are 30 students left.	I used place value blocks or unifix cubes to build towers of 10. I started with 6 towered of 10 and removed 3. Had 3 towers left. 3 towers have a value of 30. There are 30 students left.	Students mentally apply their knowledge of addition to solve this subtraction problem. I know that 30 plus 30 is 60, so 60 minus 30 equals 30. There are 30 students left.	I used a number path. I started at 60 and moved back 3 tens and landed on 30. There are 30 students left.

Students may use interactive versions of models (base ten blocks,100s charts, number path, etc.) to demonstrate and justify their thinking.

Instructional Strategies: **See Grade 1.NBT.4**

Tools/Resources:
See engageNY Modules

Common Misconceptions: See 1.NBT.5

Domain: Measurement and Data (MD)

Cluster: *Measure lengths indirectly and by iterating units.*

Standard: Grade 1.MD.1

Order three objects by length; compare the lengths of two objects indirectly by using a third object.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #3**, Developing understanding of linear measurement and measuring lengths as iterating length units**.
- This cluster connects to *Describe and compare measurable attributes* in Kindergarten, and to *Measure and estimate lengths in standard unit*s and *Represent and interpret data* in Grade 2.

Explanation and Examples:

This standard calls for students to indirectly measure objects by comparing the length of two objects by using a third object as a measuring tool. This concept is referred to as *transitivity*.

Example:

Which is longer: the height of the bookshelf or the height of a desk?

In order for students to be able to compare objects, students need to understand that length is measured from one end point to another end point. They determine which of two objects is longer, by physically aligning the objects. Typical language of length includes taller, shorter, longer, and higher. When students use bigger or smaller as a comparison, they should explain what they mean by the word. Some objects may have more than one measurement of length, so students identify the length they are measuring. Both the length and the width of an object are measurements of length.

Examples for ordering:

- Order three students by their height
- Order pencils, crayons, and/or markers by length
- Build three towers (with cubes) and order them from shortest to tallest
- Three students each draw one line, then order the lines from longest to shortest

Example for comparing indirectly:

- Two students each make a dough "snake." Given a tower of cubes, each student compares his/her snake to the tower. Then students make statements such as, "My snake is longer than the cube tower and your snake is shorter than the cube tower. So, my snake is longer than your snake."

Instructional Strategies:
The measure of an attribute is a count of how many units are needed to fill, cover or match the attribute of the object being measured. Students need to understand what a unit of measure is and how it is used to find a measurement. They need to predict the measurement, find the measurement and then discuss the estimates, errors and the measuring process.

It is important for students to measure the same attribute of an object with differently sized units recognizing that different units will result in different measures.

It is beneficial to use informal units for beginning measurement activities at all grade levels because they allow students to focus on the attributes being measured. The numbers for the measurements can be kept manageable by simply adjusting the size of the units.

Experiences with informal or nonstandard units promote the need for measuring with standard units.

Measurement units share the attribute being measured. Students need to use as many copies of the length unit as necessary to match the length being measured. For instance, use large footprints with the same size as length units. Place the footprints end to end, without gaps or overlaps, to measure the length of a room to the nearest whole footprint. Use language that reflects the approximate nature of measurement, such as the length of the room is about 19 footprints.

Students need to also measure the lengths of curves and other distances that are not straight lines.

Students need to make their own measuring tools. For instance, they can place paper clips end to end along a piece of cardboard, make marks at the endpoints of the clips and color in the spaces. Students can now see that the spaces represent the unit of measure, not the marks or numbers on a ruler.

Eventually they write numbers in the center of the spaces. Encourage students not to use the end of the ruler as a starting point. Compare and discuss two measurements of the same distance, one found by using a ruler and one found by aligning the actual units end to end, as in a chain of paper clips.

Students should also measure lengths that are longer than a ruler.

It is important for students to measure the same attribute of an object with different sized units recognizing that different units will result in different measures.

Use of Transitive Property
Have students use reasoning to compare measurements indirectly, for example to order the lengths of Objects A, B and C, examine then compare the lengths of Object A and Object B and the lengths of Object B and Object C.

The results of these two comparisons allow students to use reasoning to determine how the length of Object A compares to the length of Object C. For example, to order three objects by their lengths, reason that if Object A is shorter than Object B and Object B is shorter than Object C, then Object A has to be shorter than Object C. The order of objects by their length from smallest to largest would be Object A - Object B - Object C.

Resources/Tools

See engageNY Modules

"Measurement Matters," Georgia Department of Education. Students make direct comparisons of two or more lengths, use non-standard units of length, and order lengths of objects in the classroom.

Common Misconceptions:

Some students may view the measurement process as a procedural counting task. They might count the markings on a ruler rather than the spaces between (the units of measure). Students need numerous experiences measuring lengths with student-made tapes or rulers with numbers in the center of the spaces.

Domain: Measurement and Data (MD)

Cluster: *Measure length indirectly and by iterating.*

Standard: Grade 1.MD.2

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Suggested Standards for Mathematical Practice (MP):

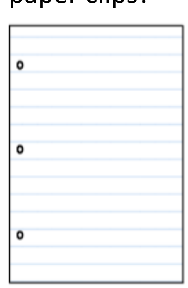
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 2.MD.1

Explanation and Examples:

Asks students to use multiple copies of one object to measure a larger object. This concept is referred to as *iteration*. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of making sure that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3rd Grade.

Example: How long is the paper in terms of paper clips?



Instructional Strategies:

Ask students to use multiple copies of one object to measure a larger object. This concept is referred to as *iteration*. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of making sure that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3rd Grade.

Students use their counting skills while measuring with non-standard units. While this standard limits measurement to whole numbers of length, in a natural environment, not all objects will measure to an exact whole unit. When students determine that the length of a pencil is six to seven paperclips long, they can state that it is about six paperclips long.

Example:

- Ask students to use multiple units of the same object to measure the length of a pencil. (How many paper clips will it take to measure how long the pencil is?)

Tools/Resources:

1.MD.A.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

- 1.MD How Long?
- 1.MD Measure Me!
- 1.MD Growing Bean Plants
- 1.OA Measuring Blocks

See: Learning Progressions for detailed information.

Common Misconceptions: See Grade 1.MD.1

Domain: Measurement and Data (MD)

Cluster: *Tell and write time.*

Standard: Grade 1.MD.3

Tell and write time in **hours** and **half-hours** using analog and digital clocks.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

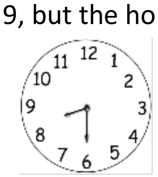
Connections:

- This cluster is connected to the First Grade Critical Area of Focus #3, **Developing understanding of linear measurement and measuring lengths as iterating length units**.
- This Cluster connects to *Work with time and money* in Grade 2.

Explanation and Examples:

This standard asks students to read both analog and digital clocks and then orally tell and write the time. Times should be limited to the hour and the half-hour.

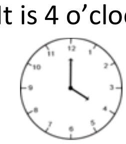
Students need experiences exploring the idea that when the time is at the half-hour the hour hand is between numbers and not on a number. Further, the hour is the number before where the hour hand is. For example, in the clock below, the time is 8:30. The hour hand is between the 8 and 9, but the hour is 8 since it is not yet on the 9.



Ideas to support telling time:

- within a day, the hour hand goes around a clock twice (the hand moves only in one direction)
- when the hour hand points exactly to a number, the time is exactly on the hour
- time on the hour is written in the same manner as it appears on a digital clock
- the hour hand moves as time passes, so when it is half way between two numbers it is at the half hour
- there are 60 minutes in one hour; so halfway between an hour, 30 minutes have passed
- half hour is written with "30" after the colon

"It is 4 o'clock"



"It is halfway between 8 o'clock and 9 o'clock. It is 8:30."

The idea of 30 being "halfway" is difficult for students to grasp. Students can write the numbers from 0 - 60 counting by tens on a sentence strip. Fold the paper in half and determine that halfway between 0 and 60 is 30. A number path on an interactive whiteboard may also be used to demonstrate this.

Instructional Strategies:
Students are likely to experience some difficulties learning about time. On an analog clock, the little hand indicates approximate time to the nearest hour and the focus is on where it is pointing. The big hand shows minutes before and after an hour and the focus is on distance that it has gone around the clock or the distance yet to go for the hand to get back to the top. It is easier for students to read times on digital clocks, but these do not relate times very well.

Students need to experience a progression of activities for learning how to tell time. Begin by using a one-handed clock to tell times in hour and half-hour intervals, then discuss what is happening to the unseen big hand. Next use two real clocks, one with the minute hand removed, and compare the hands on the clocks. Students can predict the position of the missing big hand to the nearest hour or half-hour and check their prediction using the two-handed clock. They can also predict the display on a digital clock given a time on a one- or two-handed analog clock and vice-versa.

Have students tell the time for events in their everyday lives to the nearest hour or half hour.
Make a variety of models for analog clocks. One model uses a strip of paper marked in half hours. Connect the ends with tape to form the strip into a circle.

Resources/Tools
1.MD Making a clock
"It's Time", Georgia Department of Education. Students explore when it is important to know time and view a video. Using the interactive white board, students show times to the hour on a clock and model time on digital and analog clocks.

Common Misconceptions:
The use of "paper plate clocks" is highly discouraged. These do not keep the relationship between the hour hand and the minute hand. Essentially analog clocks operate on a ratio that students will learn about in later grades. Broken clocks that no longer run can provide models in the absence of manipulative clocks.

Domain: Measurement and Data (MD)

Cluster: Represent and interpret data.

Standard: Grade 1.MD.4

Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and **how many more** or **less** are in one category than in another.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #3, **Developing understanding of linear measurement and measuring lengths as iterating length units**.
- This cluster connects to *Classify objects and counts the number of objects in each category* in Kindergarten, and to *Represent and interpret data* in Grade 2.

Explanation and Examples:

This standard calls for students to create graphs and tally charts using data relevant to their lives (e.g. **categorical data**--favorite ice cream, eye color, pets, etc). Graphs may be constructed by groups of students as well as by individual students. Then, they work with the data by organizing, representing and interpreting data. Students should have experiences posing a question with 3 possible responses and then work with the data that they collect.

Counting objects should be reinforced when collecting, representing, and interpreting data. Students describe the object graphs and tally charts they create. They should also ask and answer questions based on these charts or graphs that reinforce other mathematics concepts such as sorting and comparing. The data chosen or questions asked give students opportunities to reinforce their understanding of place value, identifying ten more and ten less, relating counting to addition and subtraction and using comparative language and symbols.

Example:

Students pose a question and the 3 possible responses.

Which is your favorite flavor of ice cream? Chocolate, vanilla or strawberry?

Students collect their data by using tallies or another way of keeping track.

Students organize their data by totaling each category in a chart or table.

What is your favorite flavor of ice cream?	
Chocolate	12
Vanilla	5
Strawberry	6

Students interpret the data by comparing categories.

Examples of comparisons:

What does the data tell us? Does it answer our question?

- More people like chocolate than the other two flavors.
- Only 5 people liked vanilla.
- Six people liked Strawberry.
- 7 more people liked Chocolate than Vanilla.
- The number of people that liked Vanilla was 1 less than the number of people who liked Strawberry.
- The number of people who liked either Vanilla or Strawberry was 1 less than the number of people who liked chocolate.
- 23 people answered this question.

Instructional Strategies:

Ask students to sort a collection of items in up to three categories. Then ask questions about the number of items in each category and the total number of items. Also ask students to compare the number of items in each category. The total number of items to be sorted should be less than or equal to 100 to allow for sums and differences less than or equal to 100 using the numbers 0 to 100.

Connect to the geometry content studied in Grade 1. Provide categories and have students sort identical collections of different geometric shapes. After the shapes have been sorted, ask these questions: How many triangles are in the collection? How many rectangles are there? How many triangles and rectangles are there? Which category has the most items? How many more? Which category has the least? How many less?

Students can create real or cluster graphs after they have had multiple experiences with sorting objects according to given categories. The teacher should model a cluster graph several times before students make their own. A cluster graph in Grade 1 has two or three labeled loops or regions (categories). Students place items inside the regions that represent a category that they chose. Items that do not fit in a category are placed outside of the loops or regions. Students can place items in a region that overlaps the categories if they see a connection between categories. Ask questions that compare the number of items in each category and the total number of items inside and outside of the regions.

Resources/Tools

1, 2.MD Favorite Ice Cream Flavor
1.MD Weather Graph Data
"Jelly Bean Graph", Georgia Department of Education.

Students make several grabs of jelly beans and record the information on a graphic organizer and decide if the number is more than, less than, or equal to ten and record it on their tally sheets. They discuss how to create a graph to show results and write questions to help others interpret their graphs.

Common Misconceptions:

Students may think that graphs are "pictures" of situations, rather than abstract representations.

Domain: Geometry (G)

***Cluster:** Reason with shapes and their attributes*.

Standard: Grade 1.G.1

Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

- This cluster is connected to the First Grade Critical Area of Focus #4**, Reasoning about attributes of, and composing and decomposing geometric shapes**.
- This cluster is connected to both clusters in the Geometry Domain in Kindergarten and *to Reason with shapes and their attributes* in Grade 2.

Explanation and Examples:

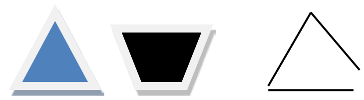
Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

This standard calls for students to determine which attributes of shapes are defining compared to those that are non-defining. Defining attributes are attributes that must always be present. Non-defining attributes are attributes that do not always have to be present. The shapes can include triangles, squares, rectangles, and trapezoids.

Asks students to determine which attributes of shapes are defining compared to those that are non-defining. Defining attributes are attributes that help to define a particular shape (#angles, # sides, length of sides, etc.). Non-defining attributes are attributes that do not define a particular shape (color, position, location, etc.). The shapes can include triangles, squares, rectangles, and trapezoids. 1.G.2 includes half-circles and quarter-circles.

Example:
All triangles must be closed figures and have 3 sides. These are defining attributes.
Triangles can be different colors, sizes and be turned in different directions, so these are non-defining.

Student 1
Which figure is a triangle?
How do you know that it is a triangle?
"It has 3 sides. It's also closed."



Attributes refer to any characteristic of a shape. Students use attribute language to describe a given two-dimensional shape: number of sides, number of vertices/points, straight sides, closed. A child might describe a triangle as "right side up" or "red." These attributes are not defining because they are not relevant to whether a shape is a triangle or not. Students should articulate ideas such as, "A triangle is a triangle because it has three straight sides and is closed."

It is important that students are exposed to both regular and irregular shapes so that they can communicate defining attributes. Students should attend to precision and use attribute language to describe why these shapes are not triangles.

Students should also use appropriate language to describe a given three-dimensional shape: number of faces, number of vertices/points, number of edges.

Example: A cylinder may be described as a solid that has two circular faces connected by a curved surface (which is not considered a face). Students may say, "It looks like a can."

Students should compare and contrast two-and three-dimensional figures using defining attributes.

Examples:
- List two things that are the same and two things that are different between a triangle and a cube.
- Given a circle and a sphere, students identify the sphere as being three-dimensional but both are round.
- Given a trapezoid, find another two-dimensional shape that has two things that are the same.

Instructional Strategies: **(1.G.1-3)**
Students can easily form shapes on geoboards using colored rubber bands to represent the sides of a shape. Ask students to create a shape with four sides on their geoboard, and then copy the shape on dot paper. Students can share and describe their shapes as a class while the teacher records the different defining attributes mentioned by the students.

Pattern block pieces can be used to model defining attributes for shapes. Ask students to create their own rule for sorting pattern blocks. Students take turns sharing their sorting rules with their classmates and showing examples that support their rule. The classmates then draw a new shape that fits this same rule after it is shared.

Students can use a variety of manipulatives and real-world objects to build larger shapes. The manipulatives can include paper shapes, pattern blocks, color tiles, triangles cut from squares (isosceles right triangles), tangrams, canned food (right circular cylinders) and gift boxes (cubes or right rectangular prisms).

Folding shapes made from paper enables students to physically feel the shape and form the equal shares. Ask students to fold circles and rectangles first into halves and then into fourths. They should observe and then discuss the change in the size of the parts.

Students may use interactive whiteboards or computer environments to move shapes into different orientations and to enlarge or decrease the size of a shape still keeping the same shape. They can also move a point/vertex of a triangle and identify that the new shape is still a triangle. When they move one point/vertex of a rectangle they should recognize that the resulting shape is no longer a rectangle.

Resources/Tools

1.G 3-D Shape Sort

1.G All vs. Only some

”Which One Doesn’t Belong”, Georgia Department of Education.

Students become familiar with the different ways to classify shapes by attributes (size, shape, color, number of sides)and self-select multiple shapes and classify them by a common attribute, and determine “which one doesn’t belong”.

Common Misconceptions:

Students may think that a square that has been rotated 45-degree is no longer a square but a diamond. They need to have experiences with shapes in different orientations. For example, in *building-shapes*, ask students to orient the smaller shapes in different ways.

Some students may think that the size of the equal shares is directly related to the number of equal shares. For example, they think that fourths are larger than halves because there are four fourths in one whole and only two halves in one whole. Students need to focus on the change in the size of the fractional parts as recommended in the folding shapes strategy. (Focus on Concrete and Representational activities).

Domain: Geometry (G)

***Cluster:** Reason with shapes and their attributes.*

Standard: Grade 1.G.2

Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.*

***Students do not need to learn formal names such as "right rectangular prism."**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 1.G.1

Explanation and Examples:

In this standard students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.
This standard calls for students to compose (build) a two-dimensional or three-dimensional shape from two shapes. This standard includes shape puzzles in which students use objects (e.g., pattern blocks) to fill a larger region.

The ability to describe, use and visualize the effect of composing and decomposing shapes is an important mathematical skill. It is not only relevant to geometry, but is related to children's ability to compose and decompose numbers.

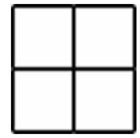
Examples:

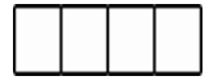
- Show the different shapes that you can make by joining a triangle with a square.
- Show the different shapes you can make joining a trapezoid with a half-circle.
- Show the different shapes you can make with a cube and a rectangular prism.

Students may use pattern blocks, plastic shapes, tangrams, or computer environments to make new shapes. The teacher can provide students with cutouts of shapes and ask them to combine them to make a particular shape.

Example:

- What shapes can be made from four squares?





Students can make three-dimensional shapes with clay or dough, slice into two pieces (not necessarily congruent) and describe the two resulting shapes. For example, slicing a cylinder will result in two smaller cylinders.

Instructional Strategies: **See Grade 1.G.1**

Resources/Tools
1.G Overlapping Rectangles
1.G Counting Squares
1.G Make Your Own Puzzle
1.G Grandfather Tang's Story

Common Misconceptions: See Grade 1.G.1

Domain: Geometry (G)

Cluster: *Reason with shapes and their attributes.*

Standard: Grade 1.G.3

Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 1.G.1

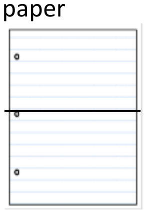
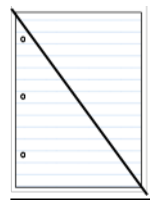
Explanation and Examples:

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

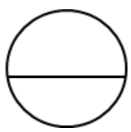
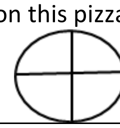
This is the first time students begin partitioning regions into equal shares using a context such as cookies, pies, pizza, etc. This is a foundational building block of fractions, which will be extended in future grades. Students should have ample experiences using the words, *halves, fourths,* and *quarters*, and the phrases *half of, fourth of,* and *quarter of*. Students should also work with the idea of the whole, which is composed of two halves, or four fourths or four quarters.

Example:

How can you and a friend share equally (partition) this piece of paper so that you both have the same amount of paper to paint a picture?

Student 1:	**Student 2**:
I would split the paper right down middle. That gives me 2 halves. I have half of the paper and my friend has the other half of the paper 	I would split it from corner to corner (diagonally). She gets half of the paper and I get half of the paper. See, if we cut here (along the line) the parts are the same size. 

Example:

Teacher:	**Teacher**:
There is pizza for dinner. What do you notice about the slice of the pizza? 	If we cut the same pizza into four slices (fourths) do you think the slices would be the same size, larger or smaller as the slices on this pizza? 

Student:	**Student**:
There are two slices on the pizza. Each slice is the same size. Those are big slices.	*When you cut the pizza into fourths. The slices are smaller than the other pizza. More slices mean that the slices get smaller. I want a slice from the first pizza.*

Students need many experiences with different sized circles and rectangles to recognize that when they cut something into two equal pieces, each piece will equal one half of its original whole. Children should recognize that halves of two different wholes are not necessarily the same size. Also they should reason that decomposing equal shares into more equal shares results in smaller equal shares.

More Examples:

- Student partitions a rectangular candy bar to share equally with one friend and thinks “I cut the rectangle into two equal parts. When I put the two parts back together, they equal the whole candy bar. One half of the candy bar is smaller than the whole candy bar.”

- Student partitions an identical rectangular candy bar to share equally with 3 friends and thinks “I cut the rectangle into four equal parts. Each piece is one fourth of or one quarter of the whole candy bar. When I put the four parts back together, they equal the whole candy bar. I can compare the pieces (one half and one fourth) by placing them side-by-side. One fourth of the candy bar is smaller than one half of the candy bar.

- Students partition a pizza to share equally with three friends. They recognize that they now have four equal pieces and each will receive a fourth or quarter of the whole pizza.

Instructional Strategies: **See Grade 1.G.1**

Resources/Tools

[Modules (engageNY)]

Common Misconceptions: See Grade 1.G.1

Appendix

TABLE 1. Common Addition and Subtraction Situations[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ or $5 = 5 + 0$ $5 = 1 + 4$ or $5 = 4 + 1$ $5 = 2 + 3$ or $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$

[1]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common Multiplication and Division Situations[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ And $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*: You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*: You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The Properties of Operations

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every (a) there exists$(-a)$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

*TABLE 4. The Properties of Equality**

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$ then $b=a$
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$
Addition property of equality	If $a=b$ then $a+c=b+c$
Subtraction property of equality	If $a=b$ then $a-c=b-c$
Multiplication property of equality	If $a=b$ then $a\times c=b\times c$
Division property of equality	If $a=b$ and $c\neq 0$ then $a\div c=b\div c$
Substitution property of equality	If $a=b$ then b may be substituted for a in any expression containing a.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.
If $a > b$ and $b > c$ then $a > c$
If $a > b$ then $b < a$
If $a > b$ then $-a < -b$
If $a > b$ then $a \pm c > b \pm c$
If $a > b$ and $c > 0$ then $a \times c > b \times c$
If $a > b$ and $c < 0$ then $a \times c < b \times c$
If $a > b$ and $c > 0$ then $a \div c > b \div c$
If $a > b$ and $c < 0$ then $a \div c < b \div c$

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All a: 1 2 3 4 5 6 7 8 b: 1 2 3 4 5 6 ○○○○○ ○○○ ○○○○○ ○ c: 1 2 3 4 5 6 7 8 9 10 11 12 13 14	Take Away a: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 ~~○○○○○ ○○○~~○○ ○○○○ b: 1 2 3 4 5 6 7 8 c: 1 2 3 4 5 6
Level 2: Count on	Count On 8 (○○○○○ ○○○) → 8 ○○○○○ ○ 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 I took away 8 9 10 11 12 13 14 8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend	Recompose: Make a Ten (○○○○○ ○○○ ○○)○○○○ ○ 10 + 4	14 – 8: I make a ten for 8 + ? = 14 ~~○○○○○ ○○○~~○○ ○○○○ 8 + 2 + 4 6
Make a ten (from 5's within each addend)	(○○○○○) (○○○) (○○○○○)(○) 10 + 4	8 + 6 = 14
Doubles ± *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words.

Table 7: Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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