Common Core State Standards for Mathematics

Flip Book

Grade 8

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This project used the work done by the Departments of Educations in Ohio, North Carolina, Georgia, engageNY, NCTM, and the Tools for the Common Core Standards.

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About the Flip Books

The development of the “flip books” is in response to the adoption of the Common Core State Standards by the state of Kansas in 2010. Teachers who were beginning the transition to the new Kansas Standards—Kansas College and Career Ready Standards (KCCRS) needed a reliable starting place that contained information and examples related to the new standards.

This project attempts to pull together, in one document some of the most valuable resources that help develop the intent, the understanding and the implementation of the KCCRS. The intent of these documents is to provide a starting point for teachers and administrators to begin unraveling the standard and is by no means the only necessary or complete resource that supports implementation of KCCRS.

This project began in the summer 2012 with the work of Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the “flip books”. The “flip books” are based on a model that Kansas had for earlier standards however, this edition is far more comprehensive than those in the past. The current editions incorporate the resources from: other state departments of education, documents such as the content progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. The current product was a compilation of work from the project developers in conjunction with many mathematics educators from around the state. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KATM website at www.katm.org and will continue to undergo changes periodically. When significant changes/additions are implemented the necessary modification will be posted and dated.

The initial development of the current update to the “flip books” was driven by the need expressed by teachers of mathematics in Kansas and with the financial support from Kansas Department of Education and encouragement from the Kansas Association of Teachers of Mathematics. These “flip books” have become an ongoing resource that will continue to evolve as more is learned about high quality instruction for the KCCRS for mathematics.

For questions or comments about the flipbooks please contact Melisa Hancock at melisa@ksu.edu.

Planning Advice--Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptually understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **<u>priorities</u>** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in "Additional and Sample clusters". Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with "+" within the standards. These standards fall outside of priority recommendations.

Recommendations for using cluster level priorities

<u>Appropriate Use:</u>

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

<u>Things to Avoid:</u>

- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters' headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.

DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus**, **rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. See Table 6 Appendix, Depth of Knowledge (DOK)

In this document, Depth Opportunities are highlighted pink in the Standards section. For example:

5.NBT.6 *Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.*

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Grade 8

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 5 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They solve real world problems through application of algebraic and geometric concepts. They see the meaning of a problem and look for efficient ways to represent and solve it. They check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?” They understand the approaches of others to solving complex problems and identify correspondences between the different approaches. Example: to understand why a 20% discount followed by a 20% markup does not return an item to its original price, a MS student might translate the situation into a tape diagram or a general equation; or they might first consider the result for an item priced at $1.00 or $10.00.
2) Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. They *contextualize* to understand the meaning of the number or variable as related to the problem. They *decontextualize* to manipulate symbolic representations by applying properties of operations. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. Examples: 1)They apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems, 2) they solve problems involving unit rates by representing the situations in equation form, and 3) they use properties of operation to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plot, dot plots, histograms, etc.) Example: Use of numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that 5 – 2x is equivalent to 3x. Proficient MS students progress from arguing exclusively through concrete referents such as physical objects and pictorial referents, to also including symbolic representations such as expressions and equations.

4) Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They analyze relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the mode if it has not served its purpose. Examples: MS students might apply proportional reasoning to plan a school event or analyze a problem in the community, or they can roughly fit a line to a scatter plot to make predictions and gather experimental data to approximate a probability.
5) Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software. They are sufficiently familiar with tools appropriate for their grade to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. Examples: Use graphs to model functions, algebra tiles to see how properties of operations apply to equations, and dynamic geometry software to discover properties of parallelograms.
6) Attend to precision.	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussions with others and in their *own* reasoning. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Examples: 1) MS students can use the definition of rational numbers to explain why a number is irrational, and describe congruence and similarity in terms of transformations in the plane and 2) they accurately apply scientific notation to large numbers and use measures of center to describe data sets.
7) Look for and make use of structure.	Mathematically proficient students look for and notice patterns and then articulate what they *see*. They can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. Examples: 1) MS students might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see the equation $3x = 2y$ represents a proportional relationship with a unit rate of 3/2 = 1.5, 2) they might recognize how the Pythagorean theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. By paying attention to the calculation of slope as they repeatedly check whether po9ints are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. Examples: 1)By working with tables of equivalent ratios, middle school students can deduce the corresponding multiplicative relationships and connections to unit rates, 2) they notice the regularity with which interior angle sums increase with the number of sides in a polygon leads to a general formula for the interior angle sum of an *n*-gon, 3) MS students learn to see subtraction as addition of opposite, and use this in a general purpose tool for collecting like terms in linear expressions.

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. **Make sense of problems and persevere in solving them.** • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; "Does this make sense?"	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
2. **Reason abstractly and quantitatively.** • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is ______related to ______? • What is the relationship between _______and ______? • What does _______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
3. **Construct viable arguments and critique the reasoning of others.** • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? How can we be sure that _________? / How could you prove that._____? Will it still work if._____? • What were you considering when._______? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about._______? • How could you demonstrate a counter-example?
4. **Model with mathematics.** • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. **Use appropriate tools strategically.** • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative? • Why was it helpful to use.______? • What can using a _____ show us, that ____may not? • In what situations might it be more informative or helpful to use.______?
6. **Attend to precision.** • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Is there a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties can you use to explain.____? • How could you test your solution to see if it answers the problem?
7. **Look for and make use of structure.** • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	• What observations do you make about._____? • What do you notice when._____? • What parts of the problem might you eliminate, simplify? • What patterns do you find in._____? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to._______? • In what ways does this problem connect to other mathematical concepts?
8. **Look for and express regularity in repeated reasoning.** • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Will the same strategy work in other situations? • Is this always true, sometimes true or never true? • How would we prove that._________? • What do you notice about._________? • What is happening in this situation? • What would happen if.________? • What Is there a mathematical rule for.______? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Critical Areas for Mathematics in 8th Grade

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions $\left(\frac{y}{x} = m \text{ or } y = mx\right)$ as special linear equations $(y = mx + b)$, understanding that the constant of proportionality (*m*) is the slope, and the graphs are lines through the origin. They understand that the slope (*m*) of a line is a constant rate of change, so that if the input or *x*-coordinate changes by an amount *A*, the output or *y*-coordinate changes by the amount *m*• *A*. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and *y*-intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres..

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Grade 8 Content Standards Overview

The Number System (NS)

- Know that there are numbers that are not rational, and approximate them by rational numbers.
  NS.1 **NS.2**

Expressions and Equations (EE)

- Work with radicals and integer exponents.
  EE.1 **EE.2** **EE.3** **EE.4**
- Understand the connections between proportional relationships, lines, and linear equations.
  EE.5 **EE.6**
- Analyze and solve linear equations and pairs of simultaneous linear equations.
  EE.7 **EE.8**

Functions (F)

- Define, evaluate, and compare functions.
  F.1 **F.2** **F.3**
- Use functions to model relationships between quantities.
  F.4 **F.5**

Geometry (GE)

- Understand congruence and similarity using physical models, transparencies, or geometry software.
  G.1 **G.2** **G3.** **G.4** **G.5**
- Understand and apply the Pythagorean Theorem.
  G.6 **G.7** **G.8**
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
  G.9

Statistics and Probability (SP)

- Investigate patterns of association to bivariate data.
  SP.1 **SP.2** **SP.3** **SP.4**

Major Supporting Additional **Depth Opportunities(DO)**

Domain: The Number System (NS)

Cluster: *Know that there are numbers that are not rational, and approximate them by rational numbers.*

Standard: Grade 8.NS.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: **Grade 8.EE.4; Grade 8.EE.7b**

This cluster is connected to:

- This cluster goes beyond the Grade 8 Critical Areas of Focus to address **working with irrational numbers, integer exponents, and scientific notation**.
- This cluster builds on previous understandings from Grades 6-7, The Number System.

Explanations and Examples:

Students distinguish between rational and irrational numbers. Any number that can be expressed as a fraction is a rational number. Students recognize that the decimal equivalent of a fraction will either terminate or repeat. Fractions that terminate will have denominators containing only prime factors of 2 and/or 5. This understanding builds on work in 7th grade when students used long division to distinguish between repeating and terminating decimals. Students convert repeating decimals into their fraction equivalent using patterns or algebraic reasoning. One method to find the fraction equivalent to a repeating decimal is shown below.

Change $0.\overline{4}$ to a fraction

- Let $x = 0.4444444 \ldots$
- Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving $10x = 4.4444444 \ldots$
- Subtract the original equation from the new equation.

$$10x = 4.4444444 \ldots$$
$$x = 0.44444 \ldots$$
$$9x = 4$$

- Solve the equation to determine the equivalent fraction.

$$\frac{9x}{9} = \frac{4}{9}$$
$$x = \frac{4}{9}$$

Additionally, students can investigate repeating patterns that occur when fractions have a denominator of 9, 99, or 11. For example, $\frac{4}{9}$ is equivalent to $0.\overline{4}$, $\frac{5}{9}$ is equivalent to $0.\overline{5}$, etc.

Major | Supporting | Additional | **Depth Opportunities(DO)**

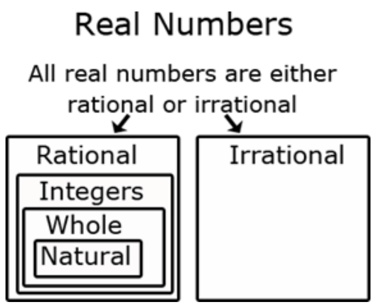
A student made the following conjecture and found two examples to support the conjecture.

> *If a rational number is not an integer, then the square root of the rational number Is irrational. For example,* $\sqrt{3.6}$ *is irrational and* $\sqrt{\frac{1}{2}}$ is irrational.

Provide two examples of non-integer rational numbers that show that the conjecture is **false.**

Sample Response:
- Example 1: 2.25
- Example 2: $\frac{1}{4}$

Students can use graphic organizers to show the relationship between the subsets of the real number system.

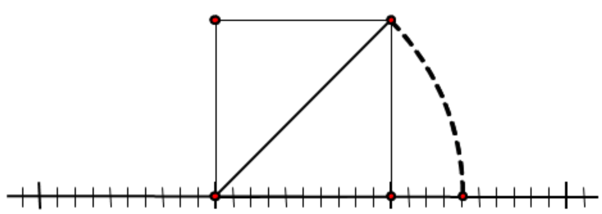


Instructional Strategies:

The distinction between rational and irrational numbers is an abstract distinction, originally based on ideal assumptions of perfect construction and measurement. In the real world, however, all measurements and constructions are approximate. Nonetheless, it is possible to see the distinction between rational and irrational numbers in their decimal representations.

A rational number is of the form $\frac{a}{b}$, where *a* and *b* are both integers, and *b* is not 0. In the elementary grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into *b* equal parts; then, beginning at 0, count out a of those parts. The surprising fact, now, is that there are numbers on the number line that cannot be expressed as $\frac{a}{b}$, with *a* and *b* both integers, and these are called irrational numbers.

Students construct a right isosceles triangle with legs of 1 unit. Using the Pythagorean Theorem, they determine that the length of the hypotenuse is $\sqrt{2}$. In the figure below, they can rotate the hypotenuse back to the original number line to show that indeed $\sqrt{2}$ is a number on the number line.



In the elementary grades, students become familiar with decimal fractions, most often with decimal representations that terminate a few digits to the right of the decimal point. For example, to find the exact decimal representation of $\frac{2}{7}$, students might use their calculator to find $\frac{2}{7} = 0.2857142857\ldots$ and they might guess that the digits 285714 repeat. To show that the digits do repeat, students in Grade 7 actually carry out the long division and recognize that the remainders repeat in a predictable pattern—a pattern that creates the repetition in the decimal representation (see 7.NS.2.d).

Thinking about long division ask students what will happen if the remainder is 0 at some step. They can reason that the long division is complete, and the decimal representation terminates. If the reminder is never 0, in contrast, then the remainders will repeat in a cyclical pattern because at each step with a given remainder, the process for finding the next remainder is always the same. Thus, the digits in the decimal representation also repeat. When dividing by 7, there are 6 possible nonzero remainders, and students can see that the decimal repeats with a pattern of at most 6 digits. In general, when finding the decimal representation of $\frac{m}{n}$, students can reason that the repeating portion of decimal will have at most *n*-1 digits. The important point here is that students can see that the pattern will repeat, so they can imagine the process continuing without actually carrying it out.

Conversely, given a repeating decimal, students learn strategies for converting the decimal to a fraction. One approach is to notice that rational numbers with denominators of 9 repeat a single digit. With a denominator of 99, two digits repeat; with a denominator of 999, three digits repeat, and so on.

- $\frac{13}{99} = 0.13131313\ldots$
- $\frac{74}{99} = 0.74747474\ldots$
- $\frac{237}{999} = 0.237237237\ldots$
- $\frac{485}{999} = 0.485485485\ldots$

From this pattern, students can go in the other direction, conjecturing, for example, that the repeating decimal $0.285714285714\ldots = \frac{285714}{999999}$. And then they can verify that this fraction is equivalent to $\frac{2}{7}$.

Once students understand that (**1**) every rational number has a decimal representation that either terminates or repeats, and (**2**) every terminating or repeating decimal is a rational number, they can reason that on the number line, irrational numbers (i.e., those that are not rational) must have decimal representations that neither terminate nor repeat. Although students at this grade do not need to be able to prove that $\sqrt{2}$ is irrational, they need to know that $\sqrt{2}$ is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats.

Nonetheless, they can approximate $\sqrt{2}$ without using the square root key on the calculator.

Students can create tables like those shown to approximate $\sqrt{2}$ to one, two, and then three places to the right of the decimal point:

x	x^2
1.0	1.00
1.1	1.21
1.2	1.44
1.3	1.69
1.4	1.96
1.5	2.25
1.6	2.56
1.7	2.89
1.8	3.24
1.9	3.61
2.0	4.00

x	x^2
1.40	1.9600
1.41	1.9881
1.42	2.0164
1.43	2.0449
1.44	2.0736
1.45	2.1025
1.46	2.1316
1.47	2.1609
1.48	2.1904
1.49	2.2201
1.50	2.2500

x	x^2
1.410	1.988100
1.411	1.990921
1.412	1.993744
1.413	1.996569
1.414	1.999396
1.415	2.002225
1.416	2.005056
1.417	2.007889
1.418	2.010724
1.419	2.013561
1.420	2.016400

From knowing that $1^2 = 1$ and $2^2 = 4$, or from the tables on the previous page, students can reason that there is a number between 1 and 2 whose square is 2. In the first table above, students can see that between 1.4 and 1.5, there is a number whose square is 2. Then in the second table, they locate that number between 1.41 and 1.42. And in the third table they can locate $\sqrt{2}$ between 1.414 and 1.415.

Students can develop more efficient methods for this work. For example, from the picture above, they might have begun the first table with 1.4. And once they see that 1.422 > 2, they do not need to generate the rest of the data in the second table.

Use set diagrams to show the relationships among real, rational, irrational numbers, integers, and counting numbers. The diagram should show that the all real numbers (numbers on the number line) are either rational or irrational.

Given two distinct numbers, it is possible to find both a rational and an irrational number between them.

Resources/Tools:
For detailed information see Learning Progressions for The Number System 6-8:
http://commoncoretools.me/wp-content/uploads/2013/07/ccssm_progression_NS+Number_2013-07-09.pdf

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.NS Estimating Square Roots
8-NS Calculating and Rounding Numbers
N-RN, 8-NS Calculating the square root of 2
8.NS Converting Decimal Representations of Rational Numbers to Fraction Representations
8.NS Identifying Rational Numbers
8.NS Converting Repeating Decimals to Fractions

Common Misconceptions:

Some students are surprised that the decimal representation of pi does not repeat. Some students believe that if only we keep looking at digits farther and farther to the right, eventually a pattern will emerge.

A few irrational numbers are given special names (*pi* and *e*), and much attention is given to $\sqrt{2}$. Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers, in the sense that they are —denser in the real line.

Students may think that the number line only has the numbers that are labeled.

Students may confuse the radical sign with the division sign.

Students may forget that each rational number has a negative square root, as well as a principal (positive) square root.

Domain: The Number System (NS)

Cluster: *Know that there are numbers that are not rational, and approximate them by rational numbers.*

Standard: Grade 8.NS.2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., x^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.4 Model with mathematics.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:
See: **Grade 8.NS. 1**; **Grade 8.G.8**; **Grade 8.G.7**

Explanations and Examples:

Students locate rational and irrational numbers on the number line. Students compare and order rational and irrational numbers. Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational. Students also recognize that square roots may be negative and written as $-\sqrt{28}$.

To find an approximation of $\sqrt{28}$, first determine the perfect squares 28 is between, which would be 25 and 36. The square roots of 25 and 36 are 5 and 6 respectively, so we know that $\sqrt{28}$ is between 5 and 6. Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27. The estimate of $\sqrt{28}$ would be 5.27 (the actual is 5.29).

Students can approximate square roots by iterative processes.

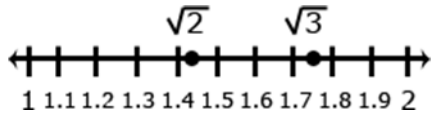
Examples:

Approximate the value of $\sqrt{5}$ to the nearest hundredth.

Solution:

Students start with a rough estimate based upon perfect squares. $\sqrt{5}$ falls between 2 and 3 because 5 falls between $2^2 = 4$ *and* $3^2 = 9$. The value will be closer to 2 than to 3. Students continue the iterative process with the tenths place value. $\sqrt{5}$ falls between 2.2 and 2.3 because 5 falls between $2.2^2 = 4.84$ *and* $2.3^2 = 5.29$. The value is closer to 2.2. Further iteration shows that the value of $\sqrt{5}$ is between 2.23 and 2.24 since 2.23^2 is 4.9729 and 2.24^2 is 5.0176

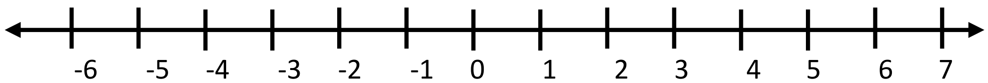
Compare $\sqrt{2}$ and $\sqrt{3}$ by estimating their values, plotting them on a number line, and making comparative statements.



Solution:
Statements for the comparison could include:
- $\sqrt{2}$ is approximately 0.3 less than $\sqrt{3}$
- $\sqrt{2}$ is between the whole numbers 1 and 2
- $\sqrt{3}$ is between 1.7 and 1.8

Without using your calculator, label approximate locations for the following numbers on the number line.

a. π
b. −(½ × π)
c. $2\sqrt{2}$
d. $\sqrt{17}$



Solution:

a. π is slightly greater than 3.
b. $-\left(\frac{1}{2} \times \pi\right)$ is slightly less than −1.5
c. $\left(2\sqrt{2}\right)^2 = 4 \cdot 2 = 8\ and\ 3^2 = 9, so\ 2\sqrt{2}$ is slightly less than 3.
d. $\sqrt{16} = 4$, so $\sqrt{17}$ is slightly greater than 4.

Instructional Strategies: See **Grade 8.NS.1**

Big ideas: Evaluate square roots and cube roots of perfect squares/cubes
Approximate square roots of non-perfect squares

Resources/Tools:

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.NS Comparing Rational and Irrational Numbers
8.NS Irrational Numbers on the Number Line
8.NS Placing a square root on the number line

Common Misconceptions: See **Grade 8.NS.1**

Domain: Expressions and Equations (EE)

Cluster: *Work with radicals and integer exponents.*

Standard: **Grade 8.EE.1**

Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3}^3 = \frac{1}{27}$.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- This cluster goes beyond the Grade 8 Critical Areas of Focus to address **working with irrational numbers, integer exponents, and scientific notation.**
- This cluster connects to previous understandings of place value, very large and very small numbers.

Explanations and Examples:

Integer (positive and negative) exponents are further used to generate equivalent numerical expressions when multiplying, dividing or raising a power to a power. Using numerical bases and the laws of exponents, students generate equivalent expressions.

Examples:

- $\frac{4^3}{5^2} = \frac{64}{25}$
- $\frac{4^3}{4^7} = 4^{3-7} = 4^{-3} = \frac{1}{4^4} = \frac{1}{256}$
- $\frac{4^{-3}}{5^2} = 4^{-3} \times \frac{1}{5^2} = \frac{1}{4^3} \times \frac{1}{5^2} = \frac{1}{64} \times \frac{1}{25} = \frac{1}{1{,}600}$
- Select all of the expressions that have a value between 0 and 1.

  A. $8^7 \cdot 8^{-12}$

  B. $\frac{7^4}{7^3}$

  C. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^9$

  D. $\frac{(-5)^6}{(-5)^{10}}$

  Solution: A, C, D

Instructional Strategies:

Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to know and use the properties of exponents and to extend the meaning beyond counting-number exponents. It is no accident that these expectations are simultaneous, because it is the properties of counting-number exponents that provide the rationale for the properties of integer exponents. In other words, s*tudents should not be told these* *properties but rather should derive them through experience and reason.*

For counting-number exponents (and for nonzero bases), the following properties follow directly from the meaning of exponents.

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$

Students should have experience simplifying numerical expressions with exponents so that these properties become natural and obvious. For example,

$2^3 \bullet 2^5 = (2\cdot2\cdot2) \bullet (2\cdot2\cdot2\cdot2\cdot2) = 2^8$
$(5^3)^4 = (5\cdot5\cdot5) \bullet (5\cdot5\cdot5) \bullet (5\cdot5\cdot5) \bullet (5\cdot5\cdot5) = 5^{12}$
$(3\cdot7)^4 = (3\cdot7) \bullet (3\cdot7) \bullet (3\cdot7) \bullet (3\cdot7) = (3\cdot3\cdot3\cdot3) \bullet (7\cdot7\cdot7\cdot7) = 3^4 \bullet 7^4$

If students reason about these examples with a sense of generality about the numbers, they begin to articulate the properties. For example, "I see that 3 twos is being multiplied by 5 twos, and the results is 8 twos being multiplied together, where the 8 is the sum of 3 and 5, the number of twos in each of the original factors. That would work for a base other than two (as long as the bases are the same)."

Note: When talking about the meaning of an exponential expression, it is easy to say (incorrectly) that "3^5 means 3 multiplied by itself 5 times."‖ But by writing out the meaning, $3^5 = 3\cdot3\cdot3\cdot3\cdot3$, students can see that there are only 4 multiplications. So a better description is "3^5 means 5 3s multiplied together."‖

Students also need to realize that these simple descriptions work only for counting- number exponents. When extending the meaning of exponents to include 0 and negative exponents, these descriptions are limiting: Is it sensible to say "3^0 means 0 3s multiplied together"‖ or that "3^{-2} means -2 3s"?

For example, Property 1 can be used to reason what 30 should be. Consider the following expression and simplification: $3^0\cdot3^5=3^{0+5} = 3^5$. This computation shows that the when 3^0 is multiplied by 3^5, the result (following Property 1) should be 3^5,which implies that 3^0 must be 1.

Because this reasoning holds for any base other than 0, we can reason that $a^0 = 1$ for any nonzero number a. To make a judgment about the meaning of 3^{-4}, the approach is similar: $3^{-4} \cdot 3^4 = 3^{-4+4} = 3^0 = 1$. This computation shows that 3^{-4} should be the reciprocal of 3^4, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that $a^{-n} = \frac{1}{a^n}$.

Properties of Integer Exponents
For any nonzero real numbers a and b and integers n and m:
1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = \frac{1}{a^n}$

Putting all of these results together, we now have the properties of integer exponents, shown in the chart.

A supplemental strategy for developing meaning for integer exponents is to make use of patterns, as shown in the chart to the right.

The meanings of 0 and negative-integer exponents can be further explored in a place-value chart:

thousands	hundreds	tens	ones		tenths	hundredths	thousandths
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
3	2	4	7	.	5	6	8

Thus, integer exponents support writing any decimal in expanded form like the following:

$3247.568=3\cdot10^3+2\cdot10^2+4\cdot10^1+7\cdot10^0+5\cdot10^{-1}+6\cdot10^{-2}+8\cdot10^{-3}$.

Patterns in Exponents

⋮	⋮
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	1/5
5^{-2}	1/25
5^{-3}	1/125
⋮	⋮

As the exponent decreases by 1, the value of the expression is divided by 5, which is the base. Continue that pattern to 0 and negative exponents.

Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. To develop familiarity, go back and forth between standard notation and scientific notation for numbers near, for example, 10^{12} or 10^{-9}. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Real-world problems can help students compare quantities and make sense about their relationship.

Provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally.

Opportunities for conceptually understanding irrational numbers should be developed. One way is for students to draw a square that is one unit by one unit and find the diagonal using the Pythagorean Theorem. The diagonal drawn has an irrational length of $\sqrt{2}$. Other irrational lengths can be found using the same strategy by finding diagonal lengths of rectangles with various side lengths.

Resources/Tools:

For detailed information see Learning Progressions for Expressions and Equations:
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.EE Extending the Definitions of Exponents, Variation 1
8.EE Ants versus humans
8.EE Raising to the zero and negative powers

Common Misconceptions:

Students may confuse the product of powers property and the power of a power property. Is $x^2 \cdot x^3$equivalent to x^5 or x^6? Students may make the relationship that in scientific notation, when a number contains one nonzero digit and a positive exponent, that the number of zeros equals the exponent.

This pattern may incorrectly be applied to scientific notation values with negative values or with more than one nonzero digit.

Students may confuse the operations for the properties of integer exponents. There is a tendency to memorize rules rather than internalize the concepts behind the laws of exponents.

Student may incorrectly assume that the value of a number is negative when its exponent is negative.

Domain: Expressions and Equations (EE)

Cluster: *Work with radicals and integer exponents.*

Standard: **Grade 8.EE.2**

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.EE.1; Grade 8.G.7; Grade 8.G.8**

Explanations and Examples:

Students recognize that squaring a number and taking the square root √ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{}$ are inverse operations.

This understanding is used to solve equations containing square or cube numbers. Equations may include rational numbers such as $x^2 = \frac{1}{4}, x^2 = \frac{4}{9}\ or\ x^3 = \frac{1}{8}$ (Note: Both the numerator and denominators are perfect squares or perfect cubes.)

Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students understand that in geometry a square root is the length of the side of a square and a cube root is the length of the side of a cube. The value of p for square root and cube root equations must be positive.

Examples:

$3^2 = 9$ and $\sqrt{9} = \pm 3$

$\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$

Solve $x^2 = 9$

Solution:

$$x^2 = 9$$
$$\sqrt{x^2} = \pm\sqrt{9}$$
$$x = \pm 3$$

Solve $x^3 = 8$

Solution:

$$x^3 = 8$$
$$\sqrt[3]{x^3} = \sqrt[3]{8}$$
$$x = 2$$

Classify the numbers in the box as perfect squares and perfect cubes. To classify a number, drag it to the appropriate column in the chart. Numbers that are neither perfect squares nor perfect cubes should **not** be placed in the chart.

1	64	96	125	200	256	333	361

Perfect Squares but Not Perfect Cubes	Both Perfect Squares and Perfect Cubes	Perfect Cubes but Not Perfect Squares

Solution:

Perfect Squares but Not Perfect Cubes	Both Perfect Squares and Perfect Cubes	Perfect Cubes but Not Perfect Squares
256 361	1 64	125

Use the numbers shown to make the equations true. Each number can be used only once.
To use a number, drag it to the appropriate box in an equation.

4	8	10	64	100	1,000

$\sqrt{\square} = \square$ $\qquad\qquad$ $\sqrt[3]{\square} = \square$

Sample Responses:

Equation 1: (64, 8) $\qquad$ Equation 1: (100,10)

Equation 2: (1000,10) $\qquad$ Equation 2: (64,4)

Ashley and Brandon have different methods for finding square roots.

Ashley's Method

To find the square root of *x,* find a number so that the product of the number and itself is *x*.

For example, $2 \bullet 2 = 4$, so the square root of 4 is 2.

Brandon's Method

To find the square root of *x*, multiply *x* by $\frac{1}{2}$. For example, $4 \cdot \frac{1}{2} = 2$, so the square root of 4 is 2.

Which student's method is **not** correct? Explain why the method you selected is **not** correct.

Sample Response:

Brandon's method is not correct.

Brandon's method works for the square root of 4, but it wouldn't work for the square root of 36. Half of 36 is 18, but the square root of 36 is 6 since 6 times 6 equals 36. Ashley describes the correct way to find the square root of a number.

Tools/Resources:

Illustrative mathematics:

Estimating Square Roots
Calculating the Square Root of 2
Placing a Square Root on the Numberline

Common Misconceptions: See **Grade 8.EE.1**

Domain: Expressions and Equations (EE)

Cluster: *Work with radicals and integer exponents.*

Standard: **Grade 8.EE.3**

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5.Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See **Grade 8.EE.1; Grade 8.G.7; Grade 8.G.8**

Explanations and Examples:

Students express numbers in scientific notation. Students compare and interpret scientific notation quantities in the context of the situation. If the exponent increases by one, the value increases 10 times. Students understand the magnitude of the number being expressed in scientific notation and choose an appropriate corresponding unit. For example, 3×10^8 is equivalent to 30 million, which represents a large quantity. Therefore, this value will affect the unit chosen.

Examples:

The average distance from Jupiter to the Sun is about 5×10^8 miles. The average distance from Venus to the Sun is about 7×10^7.

The average distance from Jupiter to the Sun is about how many times as great as the average distance from Venus to the Sun?

Solution: Any number between 7 and 7.143 inclusive.

3908 Nyx is an asteroid between Mars and Jupiter. Let *d* represent the approximate distance from **3908 Nyx** to the Sun. The average distance from Venus to the Sun is about 7×10^7. The average distance from Jupiter to the Sun is about 5×10^8 miles.

At a certain time of year, the square distance from **3908 Nyx** to the Sun is equal to the product of the average distance from Venus to the Sun and the average distance from Jupiter to the Sun.
This equation can be used to find the distance from **3908 Nyx** to the Sun, *d*, at this time of year.

$$d^2 = (7 \times 10^7)(5 \times 10^8)$$

Solve the equation for *d.* Round your answer to the nearest million.

Solution: 187,000,000

Tools/Resources:

[8.EE Ant and Elephant]

[6.EE,NS,RP; 8.EE,F Pennies to heaven]

[8.EE Orders of Magnitude]

Common Misconceptions: See **Grade 8.EE.1**

Domain: Expressions and Equations (EE)

Cluster: *Work with radicals and integer exponents.*

Standard: **Grade 8.EE.4**

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision

Connections: See **Grade 8.EE.1; Grade 8.NS.1**

Explanations and Examples:

Students use laws of exponents to multiply or divide numbers written in scientific notation. Additionally, students understand scientific notation as generated on various calculators or other technology.

Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of $2.45E + 23 \text{ is } 2.45 \times 10^{23} \text{ and } 3.5E \text{ is } 3.5 \times 10^{-4}$.

Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.

Examples:

This headline appeared in a newspaper.

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about 8×10^3 Giantburger restaurants in America.
- Each restaurant serves on average 2.5×10^3 people every day.
- There are about 3×10^8 Americans.

Explain your reasons and show clearly how you figured it out.

Sample *Response*:

If there are 8×10^3 Giantburger restaurants in America and each restaurant serves about 2.5×10^3 people every about day, then about $8 \times 10^3 \cdot 2.5 \times 10^3 = 20 \times 10^6 = 2 \times 10^7$ people eat at a Giantburger restaurant every day.

Since there are about 3×10^8 Americans, the percent of Americans who eat at a Giantburger restaurant every day can be computed by dividing the number of restaurant patrons by the total number of people

$$2 \times 10^7 \div 3 \times 10^8 = \frac{2}{3} \times 10^{-1}$$

Since $\frac{2}{3} \times 10^{-1} = \frac{2}{3} \times \frac{1}{10} = \frac{2}{30} = \frac{1}{15} = 0.066$, our estimate is that $6\frac{2}{3}\%$ of Americans eat a Giantburger restaurant every day, which is reasonably close to the claim in the newspaper.

A light-year is a unit of distance. It is the distance that light travels in 1 year. For example, the distance from the North Star to Earth is about 434 light-years because it takes light about 434 years to travel from the North Star to Earth.

The table lists five stars in the constellation Cassiopeia and their approximate distances, in light-years, from Earth.

Light travels at a speed of 3 x 108 meters per second.
Highlight each star in the table that is between 7 x 1017 meters and 3 x 1018 meters from Earth.

Click the name of a star to highlight it.

Star	Distance from Earth in Light Years
Schedar	228.56
Caph	54.46
Tsih	613.08
Ruchbah	99.41
Segin	441.95

Solution: Schedar, Ruchbah

Instructional Strategies: See **Grade 8.EE.1**

Resources/Tools:
See engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.EE Giantburgers
8.EE Ants versus humans
6.EE,NS,RP; 8.EE,F Pennies to heaven
8.EE Choosing appropriate units

Domain: Expressions and Equations (EE)

Cluster: *Understand the connections between proportional relationships, lines, and linear equations.*

Standard: Grade 8.EE.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure

Connections: Grade 8.F.2, , Grade 8.F.3; Grade8.F.2

This cluster is connected to:

- Grade 8 Critical Area of Focus #1: **formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations**
- Critical Area of Focus #3: **analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem**.

Explanations and Examples: 8.EE.5

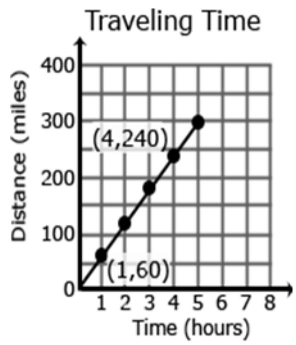
Students build on their work with unit rates from 6th grade and proportional relationships in 7th grade to compare graphs, tables and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables and equations to compare two or more proportional relationships.

Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs.

Examples:

Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

Scenario 1:



Scenario 2:

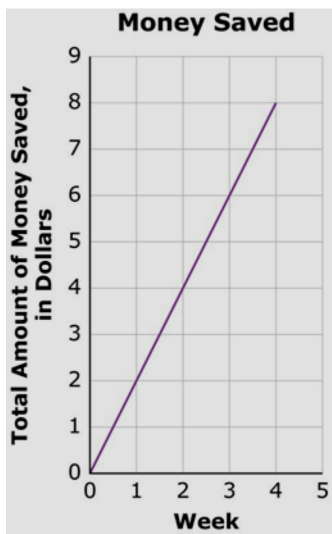
$y = 50x$

x is time in hours

y is distance in miles

Three students saved money for four weeks.

Antwan saved the same amount of money each week for 4 weeks. He made this graph to show how much money he saved.



Carla saved the same amount of money each week for 4 weeks. She made this table to show how much money she saved.

Week	Total Amount of Money Saved
1	\$1.75
2	\$3.50
3	\$5.25
4	\$7.00

Major Supporting Additional **Depth Opportunities(DO)**

Omar saved the same amount of money each week for 4 weeks. He wrote the equation below to show how much he saved. In the equation, S is the total amount of money saved, in dollars, and w is the number of weeks.

$$S = 2.5w$$

Identify the student who saved the greatest amount of money each week **and** the student who saved the least amount of money each week.

Solution:
Omar saved the greatest amount. Carla saved the least amount of money.

Instructional Strategies:

This cluster focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multistep problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed and described in different ways: graphically and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation. By using coordinate grids and various sets of three similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students can be led to generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at b.

Resources/Tools:

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

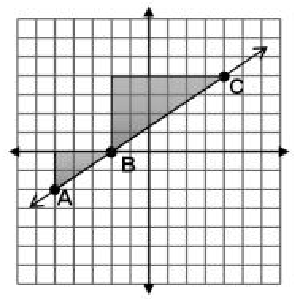
8.EE Find the Change
8.EE Equations of Lines
8.EE DVD Profits, Variation 1
8.EE Proportional relationships, lines, and linear equations
8.EE Stuffing Envelopes
8.EE Folding a Square into Thirds
8.EE Coffee by the Pound
8.EE Peaches and Plums
8.EE Who Has the Best Job?
8.EE Comparing Speeds in Graphs and Equations
8.EE Sore Throats, Variation 2
8.EE Stuffing Envelopes

Domain: Expressions and Equations (EE)

Cluster: *Understand the connections between proportional relationships, lines, and linear equations.*

Standard: **Grade 8.EE.6**

Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.



Suggested Standards for Mathematical Practice (MP):

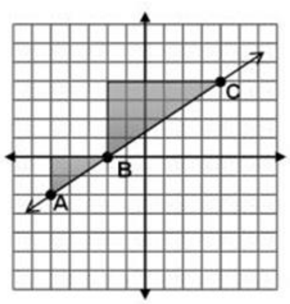
- ✓ MP.1 Solve problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 8.EE.5; Grade 8.F.3**; **Grade 8.G.4**; **Grade 8.EE.5**

Explanations and Examples:

Triangles are similar when there is a constant rate of proportion between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.

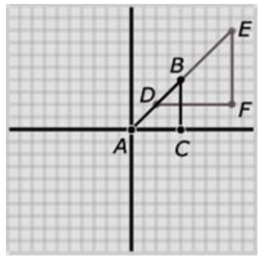
The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of $^2/_3$for the line.



Students write equations in the form $y = mx$ for lines going through the origin, recognizing that *m* represents the slope of the line. Students write equations in the form $y = mx + b$ for lines not passing through the origin, recognizing that m represents the slope and b represents the *y*-intercept.

Examples:

Explain why ΔACB is similar to ΔDFE, and deduce that $\overline{AB}$ has the same slope as $\overline{BE}$.
Express each line as an equation.



Mr. Perry's students used pairs of points to find the slopes of lines. Mr. Perry asked Avery how she used the pairs of points listed in this table to find the slope of a line.

x	*y*
8	18
20	45

Avery said, "The easiest way to find the slope is to divide *y* by *x*. The slope of this line is $\frac{18}{8}$or $\frac{9}{4}$."

Part A
Show another way to find the slope of the line that passes through the points listed in the table. Your way must be different than Avery's way.

Part B
Write an example that shows that Avery's "divide *y* by *x*" method will not work to find the slope of **any** line.

Sample Response:

Part A:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{45 - 18}{20 - 8} = \frac{27}{12} = \frac{9}{4}$$

Part B:
If Mr. Perry asked the class to find the slope of the line through (1, 1) and (2, 3), you can find the actual slope by using the formula and get $\frac{3-1}{2-1} = \frac{2}{1} = 2$, but Avery's method will not work because she would either say the slope is $\frac{1}{1} = 1$ or $\frac{3}{2} = 1.5$.

Instructional Strategies: See **Grade 8.EE.5**

Resources/Tools:

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.EE Slopes Between Points on a Line

Domain: Expressions and Equations (EE)

Cluster: *Analyze and solve linear equations and pairs of simultaneous linear equations.*

Standard: Grade 8.EE.7

Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: **Grade 8.F.3**; **Grade 8.NS.1**

This cluster is connected to:

- Grade 8 Critical Area of Focus #1: **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.**
- This cluster also builds upon the understandings in Grades 6 and 7 of Expressions and Equations, Ratios and Proportional Relationships, and utilizes the skills developed in the previous grade in The Number System.

Explanations and Examples:

Students solve one-variable equations with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property and combining like terms.

Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved to $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17):

$$10 \cdot 4 - 23 = 29 - 3 \cdot 4$$
$$40 - 23 = 29 - 12$$
$$17 = 17$$

Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal. For example, the equation $-x + 7 - 6x = 19 - 7x$, can be simplified to $-7x + 7 = 19 - 7x$. If $7x$ is added to each side, the resulting equation is $7 = 19$, which is not true. No matter what value is substituted for x the final result will be $7 = 19$. If each side of the equation were treated as a linear equation and graphed, the lines would be parallel.

An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example, the following equation, when simplified will give the same values on both sides.

$$-\frac{1}{2}(36a-6)=\frac{3}{4}(4-24a)$$
$$-18a+3=3-18a$$

If each side of the equation were treated as a linear equation and graphed, the graph would be the same line.

As students transform linear equations in one variable into simpler forms, they discover the equations can have one solution, infinitely many solutions, or no solutions.

When the equation has one solution, the variable has one value that makes the equation true as in 12-4*y*=16. The only value for y that makes this equation true is -1.

When the equation has infinitely many solutions, the equation is true for all real numbers as in
7*x* + 14 = 7 (*x*+2). As this equation is simplified, the variable terms cancel leaving 14 = 14 or 0 = 0. Since the expressions are equivalent, the value for the two sides of the equation will be the same regardless which real number is used for the substitution.

Examples:
Solve for x:

$$-3(x+7)=4$$
$$3x-8=4x-8$$
$$3(x+1)-5=3x-2$$

Solve:

$$7(m-3)=7$$
$$\frac{1}{4}-\frac{2}{3}y=\frac{3}{4}-\frac{1}{3}y$$

For each linear equation in this table, indicate whether the equation has no solution, one solution, or infinitely many solutions.

Equation	No Solution	One Solution	Infinitely Many Solutions
$7x + 21 = 21$			
$12x + 15 = 12x - 15$			
$-5x - 25 = 5x + 25$			

Solution:

1. One solution. This is designed to be an easy equation to solve to help students enter the problem. Answering this question correctly demonstrates minimal understanding.
2. No solution. Students may think there is no difference between adding 15 on the left-hand side and subtracting 15 on the right-hand side.
3. One solution. Students may think there are infinitely many solutions because the left-hand side is the negative of the right-hand side.

Three students solved the equation 3(5*x* – 14) = 18 in different ways, but each student arrived at the correct answer. Select all of the solutions that show a correct method for solving the equation.

A.

$$
\begin{aligned}
3(5x-14) &= 18\\
8x - 14 &= \ \ 18\\
+14 &\quad +14\\
\frac{8x}{8} &= \frac{32}{8}\\
x &= 4
\end{aligned}
$$

B.

$$
\begin{aligned}
\frac{1}{\cancel{3}}\cdot\cancel{3}(5x-14) &= 18\cdot\frac{1}{3}\\
5x - 14 &= \ \ 6\\
+14 &\quad +14\\
\frac{5x}{5} &= \frac{20}{5}\\
x &= 4
\end{aligned}
$$

C.

$$
\begin{aligned}
3(5x-14) &= 18\\
\frac{15x}{15} - \frac{42}{15} &= \frac{18}{15}\\
+\frac{42}{15} &\quad +\frac{42}{15}\\
x &= \frac{60}{15}\\
x &= 4
\end{aligned}
$$

Sample Response:

A. This solution is the simplest to follow, but the method is incorrect.

B. Although the method in this solution is correct, it is not the most commonly used method for solving equations like this, so students may think it is incorrect.

C. Although the method in this solution is correct, it is not the most commonly used method for solving equations like this, so students may think it is incorrect.

Consider the equation 3(2*x* + 5) = *ax* + *b*

Part A

Find one value for *a* and one value for *b* so that there is exactly one value of *x* that makes the equation true. Explain your reasoning.

Part B

Find one value for *a* and one value for *b* so that there are infinitely many values of *x* that make the equation true. Explain your reasoning.

Sample Response:

Part A

a = 5; *b* = 16 When you substitute these numbers in for *a* and *b*, you get a solution of x = 1.

Part B

a=6; *b* = 15; When you substitute these numbers in for *a* and *b*, you get a solution of 0 = 0, so there are infinitely many solutions, not just one.

Instructional Strategies:

In Grade 6, students applied the properties of operations to generate equivalent expressions, and identified when two expressions are equivalent. This cluster extends understanding to the process of solving equations and to their solutions, building on the fact that solutions maintain equality, and that equations may have only one solution, many solutions, or no solution at all. Equations with many solutions may be as simple as

$$3x = 3x, 3x + 5 = x + 2 + x + x + 3, or\ x(6 + 4)$$

where both sides of the equation are equivalent once each side is simplified.

Table 3 in the Appendix (pg.97) generalizes the properties of operations and serves as a reminder for teachers of what these properties are. Eighth graders should be able to describe these relationships with real numbers and justify their reasoning using words and not necessarily with the algebraic language of Table 3. In other words, students should be able to state that 3(-5) = (-5)3 because multiplication is commutative and it can be performed in any order (it is commutative), or that 9(8) + 9(2) = 9(8 + 2) because the distributive property allows us to distribute multiplication over addition, or determine products and add them. Grade 8 is the beginning of using the generalized properties of operations, but this is not something on which students should be assessed.

Pairing contextual situations with equation solving allows students to connect mathematical analysis with real-life events. Students should experience analyzing and representing contextual situations with equations, identify whether there is one, none, or many solutions, and then solve to prove conjectures about the solutions. Through multiple opportunities to analyze and solve equations, students should be able to estimate the number of solutions and possible values(s) of solutions prior to solving. Rich problems, such as computing the number of tiles needed to put a border around a rectangular space or solving proportional problems as in doubling recipes, help ground the abstract symbolism to life.

Experiences should move through the stages of concrete, conceptual and algebraic/abstract. Utilize experiences with the pan balance model as a visual tool for maintaining equality (balance) first with simple numbers, then with pictures symbolizing relationships, and finally with rational numbers allows understanding to develop as the complexity of the problems increases. Equation-solving in Grade 8 should involve multistep problems that require the use of the distributive property, collecting like terms, and variables on both sides of the equation.

This cluster builds on the informal understanding of slope from graphing unit rates in Grade 6 and graphing proportional relationships in Grade 7 with a stronger, more formal understanding of slope. It extends solving equations to understanding solving systems of equations, or a set of two or more linear equations that contain one or both of the same two variables. Once again the focus is on a solution to the system. Most student experiences should be with numerical and graphical representations of solutions. Beginning work should involve systems of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection, simplify the computation and hone in on finding a solution. More complex systems can be investigated and solve by using graphing technology.

Contextual situations relevant to eighth graders will add meaning to the solution to a system of equations. Students should explore many problems for which they must write and graph pairs of equations leading to the generalization that finding one point of intersection is the single solution to the system of equations. Provide opportunities for students to connect the solutions to an equation of a line, or solution to a system of equations, by graphing, using a table and writing an equation.

Instructional Strategies continued:

Students should receive opportunities to compare equations and systems of equations, investigate using graphing calculators or graphing utilities, explain differences verbally and in writing, and use models such as equation balances.

Problems such as, "Determine the number of movies downloaded in a month that would make the costs for two sites the same, when Site A charges $6 per month and $1.25 for each movie and Site B charges $2 for each movie and no monthly fee".‖

Students write the equations letting y = the total charge and x = the number of movies.

Site A: $y = 1.25x + 6$ Site B: $y = 2x$

Students graph the solutions for each of the equations by finding ordered pairs that are solutions and representing them in a t-chart. Discussion should encompass the realization that the intersection is an ordered pair that satisfies both equations. And finally students should relate the solution to the context of the problem, commenting on the practicality of their solution.

Problems should be structured so that students also experience equations that represent parallel lines and equations that are equivalent. This will help them to begin to understand the relationships between different pairs of equations: When the slope of the two lines is the same, the equations are either different equations representing the same line (thus resulting in many solutions), or the equations are different equations representing two not intersecting, parallel, lines that do not have common solutions.

System-solving in Grade 8 should include estimating solutions graphically, solving using substitution, and solving using elimination. Students again should gain experience by developing conceptual skills using models that develop into abstract skills of formal solving of equations.

Provide opportunities for students to change forms of equations (from a given form to slope- intercept form) in order to compare equations.

Resources/Tools:
See engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.EE Two Lines
8.EE The Sign of Solutions
8.EE Coupon versus discount
8.EE Solving Equations
8.EE Sammy's Chipmunk and Squirrel Observations

Common Misconceptions:
Students think that only the letters x and y can be used for variables. Students think that you always need a variable = a constant as a solution. The variable is always on the left side of the equation.

Equations are not always in the slope intercept form, $y=mx+b$. Students confuse one-variable and two-variable equations.

Domain: Expressions and Equations (EE)

Cluster: *Analyze and solve linear equations and pairs of simultaneous linear equations.*

Standard: Grade 8.EE.8

Analyze and solve pairs of simultaneous linear equations.

a. *Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.*
b. *Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
c. *Solve real-world and mathematical problems leading to two linear equations in two variables.*

For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 8.EE.7**

Explanations and Examples:

Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.

Systems of linear equations can also have one solution, infinitely many solutions or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions.

Examples:

Find *x* and *y* using elimination and then using substitution.

$$3x + 4y = 7$$
$$\underline{-2x + 8y = 10}$$

Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

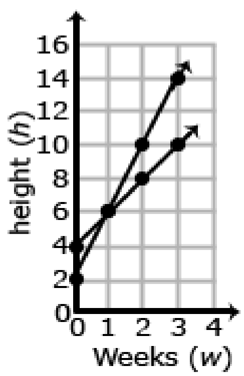
Let *W* = number of weeks
Let *H* = height of the plant after *W* weeks

Plant A		
W	**H**	
0	4	(0,4)
1	6	(1,6)
2	8	(2,8)
3	10	(3,10)

Plant B		
W	**H**	
0	2	(0,2)
1	6	(1,6)
2	10	(2,10)
3	14	(3,14)

Given each set of coordinates, graph their corresponding lines.

Solution:



Write an equation that represent the growth rate of Plant A and Plant B.

Solution:

Plant A: $H = 2W + 4$
Plant B: $H = 4W + 2$

At which week will the plants have the same height?

Solution:

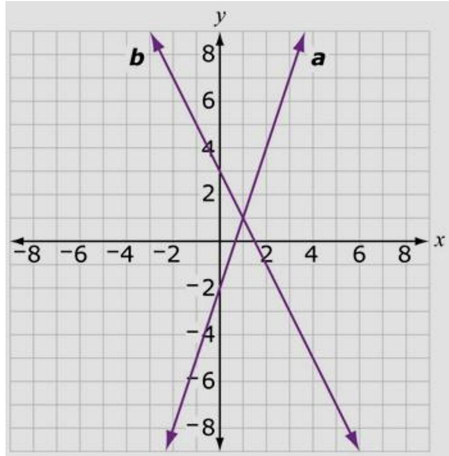
The plants have the same height after one week.

Plant A	Plant B
$H = 2W + 4$	$H = 4W + 2$
$H = 2(1) + 4$	$H = 4(1) + 4$
$H = 6$	$H = 6$

After one week, the height of Plant A and Plant B are both 6 inches.

Major Supporting Additional **Depth Opportunities(DO)**

The graphs of line *a* and line *b* are shown on this coordinate grid.



Match each line with its equation. Click on an equation and then drag it to the corresponding box for each line.

The equation of line *a* is

.

The equation of line *b* is

$y = -2x + 3$ $\quad$ $y = 2x + 3$ $\quad$ $y = 3x - 2$

$y = -\frac{1}{2}x + 3$ $\quad$ $y = -\frac{1}{3}x - 2$

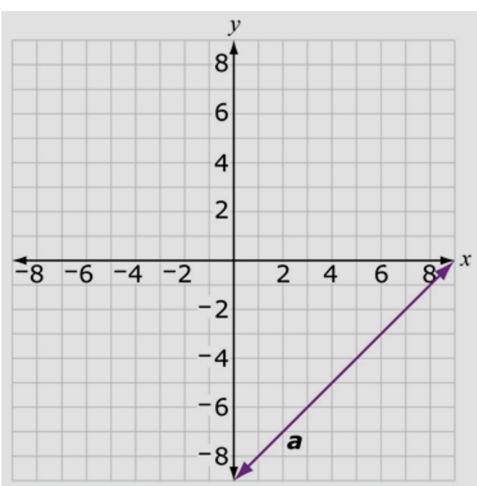
Solution: The equation of line *a* is $y = -2x + 3$.

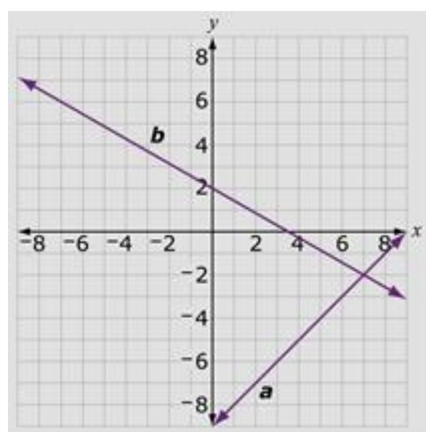
The equation of line *b* is $y = 3x - 2$.

Line *a* is shown on the coordinate grid. Construct line *b* on the coordinate grid so that

- line *a* and line *b* represent a system of linear equations with a solution of (7, –2)
- the slope of line *b* is greater than –1 and less than 0
- the *y*-intercept of line *b* is positive

Sample Response:





Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies:

This cluster builds on the informal understanding of slope from graphing unit rates in Grade 6 and graphing proportional relationships in Grade 7 with a stronger, more formal understanding of slope. It extends solving equations to understanding solving systems of equations, or a set of two or more linear equations that contain one or both of the same two variables. Once again the focus is on a solution to the system. Most student experiences should be with numerical and graphical representations of solutions. Beginning work should involve systems of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection, simplify the computation and hone in on finding a solution. More complex systems can be investigated and solve by using graphing technology.

Contextual situations relevant to eighth graders will add meaning to the solution to a system of equations. Students should explore many problems for which they must write and graph pairs of equations leading to the generalization that finding one point of intersection is the single solution to the system of equations. Provide opportunities for students to connect the solutions to an equation of a line, or solution to a system of equations, by graphing, using a table and writing an equation.

Students should receive opportunities to compare equations and systems of equations, investigate using graphing calculators or graphing utilities, explain differences verbally and in writing, and use models such as equation balances.

Problems such as, "Determine the number of movies downloaded in a month that would make the costs for two sites the same, when Site A charges \$6 per month and \$1.25 for each movie and Site B charges \$2 for each movie and no monthly fee.‖

Students write the equations letting y = the total charge and x = the number of movies.

Site A: $y = 1.25x + 6$ Site B: $y = 2x$

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Problems should be structured so that students also experience equations that represent parallel lines and equations that are equivalent. This will help them to begin to understand the relationships between different pairs of equations: When the slope of the two lines is the same, the equations are either different equations representing the same line (thus resulting in many solutions), or the equations are different equations representing two not intersecting, parallel, lines that do not have common solutions.

System-solving in Grade 8 should include estimating solutions graphically, solving using substitution, and solving using elimination. Students again should gain experience by developing conceptual skills using models that develop into abstract skills of formal solving of equations.

Provide opportunities for students to change forms of equations (from a given form to slope- intercept form) in order to compare equations.

Resources/Tools:
8.EE How Many Solutions?
8.EE Fixing the Furnace
8.EE Cell Phone Plans
8.EE Kimi and Jordan
8.EE Folding a Square into Thirds
8.EE The Intersection of Two Lines
8.EE Quinoa Pasta 1
8.EE Summer Swimming

"Cara's Candles and DVD's", Georgia Department of Education.
Students are given two tasks; both require writing two equations and solving the resulting system of equations.

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

Common Misconceptions: See **Grade 8.EE.7**

Domain: Functions (F)

Cluster: *Define, evaluate, and compare functions.*

Standard: **Grade 8.F.1**

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This Cluster is connected to:

- Grade 8 Critical Area of Focus #2: **Grasping the concept of a function and using functions to describe quantitative relationships.**
- Expressions and Equations: Linear equations in two variables can be used to define linear functions, and students can use graphs of functions to reason toward solutions to linear equations.
- Geometry: Similar triangles are used to show that the slope of a line is constant. Statistics and Probability: Bivariate data can often be modeled by a linear function.

Explanations and Examples:

Students distinguish between functions and non-functions, using equations, graphs, and tables. Non- functions occur when there is more than one *y*-value is associated with any *x*-value.

For example, the rule that takes *x* as input and gives $x^2 + 5x + 4$ as output is a function. Using y to stand for the output we can represent this function with the equation $y = x^2 + 5x + 4$, and the graph of the equation is the graph of the function. Students are not yet expected use function notation such as $f(x) = x^2 + 5x + 4$.

Examples:

Fill in each *x*-value and *y*-value in the table below to create a relation that is **not** a function.

x	y

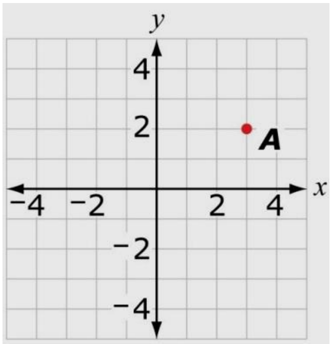
Sample Response:

x	y
4	0
4	1
4	2
4	3
4	4

Point ***A*** is plotted on the *xy*-coordinate plane below. You must determine the location of point ***C*** given the following criteria:

- Point *C* has integer coordinates.
- The graph of line $\overleftrightarrow{AC}$ is **not** a function.

Place a point on the *xy*-coordinate plane that could represent point ***C***.



Sample Responses:
(3, 5) , (3, 4) , (3, 3) , (3, 1) , (3, 0) , (3, −1) , (3, −2) , (3, −3) , (3, −4) , or (3, −5)

Instructional Strategies:

In grade 6, students plotted points in all four quadrants of the coordinate plane. They also represented and analyzed quantitative relationships between dependent and independent variables. In Grade 7, students decided whether two quantities are in a proportional relationship. In Grade 8, students begin to call relationships functions when each input is assigned to exactly one output. Also, in Grade 8, students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.

To determine whether a relationship is a function, students should be expected to **reason from a context**, a graph, or a table, after first being clear which quantity is considered the input and which is the output. When a relationship is not a function, students should produce a counterexample: an “input value”‖ with at least two “output values.”‖ If the relationship is a function, the students should explain how they verified that for each input there was exactly one output. The “vertical line test” should be avoided because (1) it is too easy to apply without thinking, (2) students do not need an efficient strategy at this point, and (3) it creates misconceptions for later mathematics, when it is useful to think of functions more broadly, such as whether *x* might be a function of *y*.

“Function machine” pictures are useful for helping students imagine input and output values, with a rule inside the machine by which the output value is determined from the input.

Notice that the standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically (symbolically, with letters). This is sometimes called the “rule of four.”‖ For fluency and flexibility in thinking, students need experiences translating among these. In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl‘s height as a function of her age.

In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe sequences both by rules relating one term to the next and also by rules for finding the *n*th term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction should focus on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively.

When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading. For example, if a function is used to model the height of a stack of n paper cups, it does not make sense to have 2.3 cups, and thus there will be no ordered pairs between $n = 2$ and $n = 3$.

Provide multiple opportunities to examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time. Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change." Compare this in order to compare two different situations and identify which is increasing/decreasing as a faster rate.

Students can compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

Resources/Tools:

For detailed information see Learning Progressions Functions:
http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

F-IF The Customers
8.F Foxes and Rabbits
8.F US Garbage, Version 1
6.EE,NS,RP; 8.EE,F Pennies to heaven
8.F Function Rules
8.F Introducing Functions

Common Misconceptions:

Some students will mistakenly think of a straight line as horizontal or vertical only.

Students may mistakenly believe that a slope of zero is the same as "no slope" and then confuse a horizontal line (slope of zero) with a vertical line (undefined slope).

Confuse the meaning of "domain" and "range" of a function.

Some students will mix up x- and y-axes on the coordinate plane, or mix up the ordered pairs. When emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up), point out that this is merely a convention: It could have been otherwise, but it is very useful for people to agree on a standard customary practice.

Domain: Functions (F)

Cluster: *Define, evaluate, and compare functions.*

Standard: Grade 8.F.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 8.F.1**; **Grade 8.EE.5**

Explanations and Examples:

Students compare functions from different representations.

For example, compare the following functions to determine which has the greater rate of change:

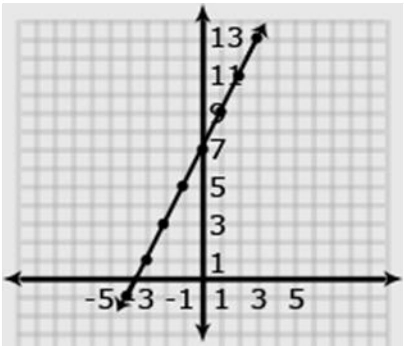
Function 1: $y = 2x + 4$

Function 2: (table)

x	***y***
-1	-6
0	-3
2	3

Examples:
Compare the two linear functions listed below and determine which equation represents a greater rate of change.

Function 1:



Function 2: The function whose input *x* and output *y* are related by: $y = 3x + 7$

Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Gift Card
Samantha starts with $20 on a gift card for the book store. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, *x*.

x	*y*
0	20
1	16.50
2	13.00
3	9.50
4	6.00

Function 2:
The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10.00 for the school year. Write the rule for the total cost (*c*) of renting a calculator as a function of the number of months (*m*).

Solution:
Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha's weekly magazine purchase.

Function 2 is an example of a function whose graph has positive slope. Students pay a yearly nonrefundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5 which is the amount of the monthly rental fee. An equation for Function 2 could be $c = 5m + 10$.

Instructional Strategies: See **Grade 8.F.1**

Resources/Tools:
[8.F Battery Charging]

Common Misconceptions: See **Grade 8.F.1**

Domain: Functions (F)

Cluster: *Define, evaluate, and compare functions.*

Standard: **Grade 8.F.3**

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.F.1**; **Grade 8.EE.5**; **Grade 8.EE7a**; **Grade 8.EE.5**; **Grade 8.EE.7**

Explanations and Examples:

Students use equations, graphs and tables to categorize functions as linear or non-linear. Students recognize that points on a straight line will have the same rate of change between any two of the points.

Examples:

Determine which of the functions listed below are linear and which are not linear and explain your reasoning.

- $y = -2x2 + 3$ non linear
- $y = 2x$ linear
- $A = \pi r2$ non linear
- $y = 0.25 + 0.5(x - 2)$ linear

Samir was assigned to write an example of a linear functional relationship. He wrote this example for the assignment.

The relationship between the year and the population of a county when the population increases by 10% each year

Part A

Complete the table below to create an example of the population of a certain county that is increasing by 10% each year.

Year	Population of a Certain County
0	
1	
2	
3	
4	

Part B
State whether Samir's example represents a linear functional relationship. Explain your reasoning.

Sample Response:

Part A

Year	Population of a Certain County
0	100,000
1	110,000
2	121,000
3	133,100
4	146,410

Part B
Samir's example is not a linear functional relationship. The population does not increase by the same amount each year, so the relationship is not linear.

Instructional Strategies: See **Grade 8.F.1**

Resources/Tools:
Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

8.F Introduction to Linear Functions

Common Misconceptions: See **Grade 8.F.1**

Domain: Functions (F)

Cluster: *Use functions to model relationships between quantities.*

Standard: **Grade 8.F.4**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: **Grade 8.SP.2**; **Grade 8.SP.3**; **Grade 8.EE.5**

This Cluster is connected to:

- Grade 8 Critical Area of Focus #2: **Grasping the concept of a function and using functions to describe quantitative relationships.**
- Expressions and Equations: Linear equations in two variables can be used to define linear functions, and students can use graphs of functions to reason toward solutions to linear equations.
- Geometry: Similar triangles are used to show that the slope of a line is constant.
- Statistics and Probability: Bivariate data can often be modeled by a linear function.

Explanations and Examples:

Students identify the rate of change (slope) and initial value (y-intercept) from tables, graphs, equations or verbal descriptions.

Students recognize that in a table the y-intercept is the y-value when x is equal to 0. The slope can the determined by finding the ratio $\frac{y}{x}$ between the change in two y-values and the change between the two corresponding x-values.

The y-intercept in the table below would be (0, 2). The distance between 8 and -1 is 9 in a negative direction is -9; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or $\frac{y}{x}$ or $\frac{-9}{3} = -3$

x	***y***
-2	8
0	2
1	-1

Using graphs, students identify the y-intercept as the point where the line crosses the y-axis and the slope as the rise, run.

In a linear equation the coefficient of x is the slope and the constant is the y-intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc. Note that point-slope form and standard forms are not expectations at this level.

In contextual situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be "converted"‖ to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

Students use the slope and y-intercepts to write a linear function in the form $y = mx + b$. Situations may be given as a verbal description, two ordered pairs, a table, a graph, or rate of change and another point on the line. Students interpret slope and y-intercept in the context of the given situation.

Examples:

The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time $25 fee for the car's navigation system (GPS). Write an expression for the cost in dollars, c, as a function of the number of days, d.

Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table.

Days (d)	**Cost (c) in dollars**
1	70
2	115
3	160
4	205

Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one time fees vs. recurrent fees will help students model contextual situations.

When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t – 100$ shows the relationship between the time of the ascent in seconds (t) and the distance from the surface in feet (d).

Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive?

Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?

You work for a video streaming company that has two plans to choose from:

Plan 1: A flat rate of \$7 per month plus \$2.50 per video viewed

Plan 2: \$4 per video viewed

a. What type of function models this situation? Explain how you know.
b. Define variables that make sense in the context and write an equation representing a function that describes each plan.
c. How much would 3 videos in a month cost for each plan? 5 videos?
d. Compare the two plans and explain what advice you would give to a customer trying to decide which plan is best for them, based on their viewing habits.

Sample Response:

a. Each plan can be modeled by a linear function since the constant rate per video indicates a linear relationship.
b. We let C_1 be the total cost per month of Plan 1, C_2 the total cost per month of Plan 2, and V the number of videos viewed in a month.
   Then $C_1(V) = 7 + 2.5V$
   $C_2(V) = 4V$
c. 3 videos on Plan 1: $C_1(3)=7+2.5(3)=\$14.50$
   5 videos on Plan 1: $C_1(5)=7+2.5(5)=\$19.50$
   3 videos on Plan 2: $C_2(3)=4(3)=\$12$
   5 videos on Plan 2: $C_2(5)=4(5)=\$20$
d. Plan 1 costs less than Plan 2 for 5 or fewer videos per month. A customer who watches more than 5 videos per month should choose Plan 2.

A customer who watches 5 or fewer videos per month should choose Plan 1.

Instructional Strategies:

In Grade 8, students focus on linear equations and functions. Nonlinear functions are used for comparison.

Students will need many opportunities and examples to figure out the meaning of $y = mx + b$.

What does m mean? What does b mean? They should be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts. For example, if a function is used to model the height of a stack of n paper cups, then the rate of change, m, which is the slope of the graph, is the height of the "lip" of the cup: the amount each cup sticks above the lower cup in the stack. The "initial value" in this case is not valid in the context because 0 cups would not have a height, and yet a height of 0 would not fit the equation. Nonetheless, the value of b can be interpreted in the context as the height of the "base" of the cup: the height of the whole cup minus its lip.

Use graphing calculators and web resources to explore linear and non-linear functions. Provide context as much as possible to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Give students opportunities to gather their own data or graphs in contexts they understand. Students need to measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They also need opportunities to draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar. Probe student thinking by asking them to determine which input values make sense in the problem situations given.

Provide students with a function in symbolic form and ask them to create a problem situation in words to match the function. Given a graph, have students create a scenario that would fit the graph. Ask students to sort a set of "cards" to match a graphs, tables, equations, and problem situations. Have students explain their reasoning to each other.

From a variety of representations of functions, students should be able to classify and describe the function as linear or non-linear, increasing or decreasing. Provide opportunities for students to share their ideas with other students and create their own examples for their classmates.

Use the slope of the graph and similar triangle arguments to call attention to not just the change in *x* or *y*, but also to the rate of change, which is a ratio of the two.

Emphasize key vocabulary. Students should be able to explain what key words mean: e.g., model, interpret, initial value, functional relationship, qualitative, linear, non-linear. Use of a “word wall” or “mind map” will help reinforce vocabulary.

Resources/Tools:

8-F Modeling with a Linear Function
8.F Heart Rate Monitoring
8.G Downhill
8.F Video Streaming
8.F High School Graduation
8.F Chicken and Steak, Variation 1
8.F Baseball Cards
8.F Chicken and Steak, Variation 2
8.F Distance across the channel
8.F Delivering the Mail, Assessment Variation

“Is it Fair?”, Georgia Department of Education.
Students play the game “Is It Fair?” and record their information using probability to determine whether they feel the game is fair or not. Predictions are made before the game begins. Based on their trials, students determine all outcomes, create tree diagrams and determine the theoretical chance of winning for each player.

See Dan Meyer Lesson “Linear Function: Stacking Cups

Common Misconceptions:

Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning—and both types of formulas—are important for developing proficiency with functions.

When input values are not increasing consecutive integers (e.g., when the input values are decreasing, when some integers are skipped, or when some input values are not integers), some students have more difficulty identifying the pattern and calculating the slope. It is important that all students have experience with such tables, so as to be sure that they do not overgeneralize from the easier examples.

Some students may not pay attention to the scale on a graph, assuming that the scale units are always "one."‖

When making axes for a graph, some students may not using equal intervals to create the scale.

Some students may infer a cause and effect between independent and dependent variables, but this is often not the case.

Some students graph incorrectly because they don't understand that *x* usually represents the independent variable and *y* represents the dependent variable. Emphasize that this is a convention that makes it easier to communicate.

Domain: Functions (F)

Cluster: *Use functions to model relationships between quantities.*

Standard: **Grade 8.F.5**

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.F.4**

Explanations and Examples:

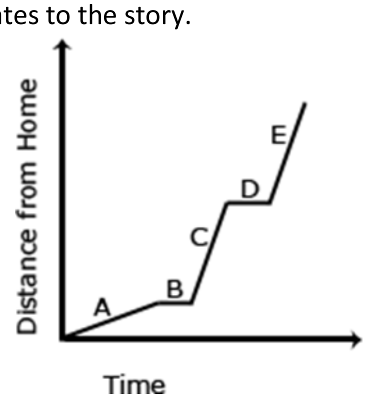
Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation.

Students learn that graphs tell stories and have to be interpreted by carefully thinking about the quantities shown.

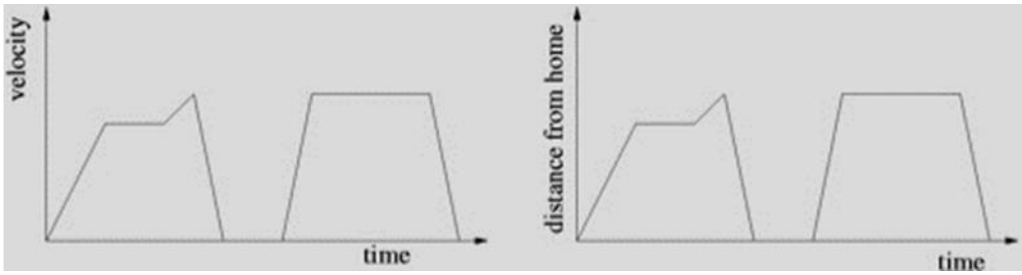
Examples:

The graph below shows a student's trip to school. This student walks to his friend's house and, together, they ride a bus to school. The bus stops once before arriving at school.

Describe how each part A-E of the graph relates to the story.



Below are two graphs that look the same. Note from the axis labels that the first graph shows the velocity of a car as a function of time and the second graph shows the distance of the car from home as a function of time. Describe what someone who observes the car's movement would see in each case.



Sample Responses:
For a velocity function, output values tell us how fast the car is moving. For the distance function, output values tell us how far from home the car is. Since we don't have scales on either axis, we can't talk about specific values of time, velocity and distance, but we can make qualitative statements about velocity and distance.

Velocity Graph: The car starts at rest and speeds up at a constant rate. When the graph becomes a horizontal line, the car is maintaining its speed for a while before speeding up for a short time and then quickly slowing down until it comes to a complete stop. It stays stationary for a little while where the graph is on the horizontal axis. Then the car speeds up, goes at a constant speed for a while and then slows down and comes to a complete stop.

Distance Graph: The car starts its trip at home. It moves away from home at a constant speed. When the graph is horizontal, the car's distance from home is not changing, which probably means it has come to a stop for awhile.* Then the car moves farther away from home before turning around and coming back home. After staying at home for a time, the car moves away from home at a constant speed. It comes to a stop for a while* before coming back home.

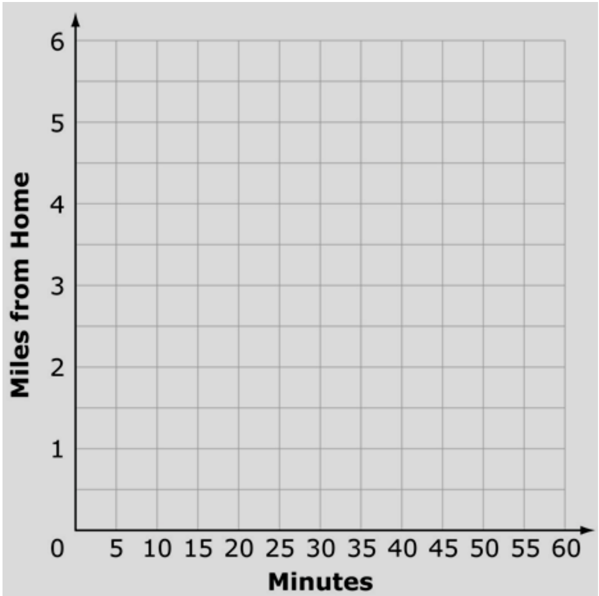
*If the distance from home is not changing, it is also possible that the car is driving along a circle with the driver's home at the center, although this doesn't seem very likely.

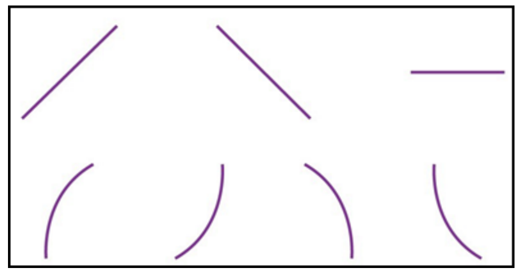
Carla rode her bike to her grandmother's house. The following information describes her trip:

- For the first 5 minutes, Carla rode fast and then slowed down. She rode 1 mile.
- For the next 15 minutes, Carla rode at a steady pace until she arrived at her grandmother's house. She rode 3 miles.
- For the next 10 minutes, Carla visited her grandmother.
- For the next 5 minutes, Carla rode slowly at first but then began to ride faster. She rode 1 mile.
- For the last 10 minutes, Carla rode fast. She rode 3 miles at a steady pace.

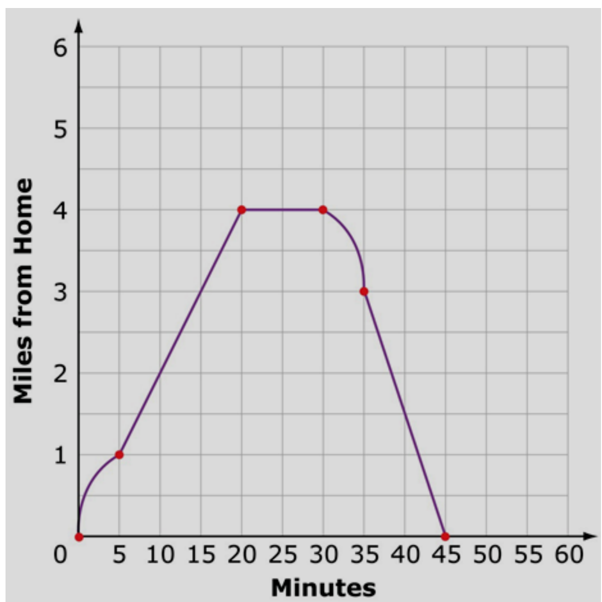
Graph each part of Carla's trip. To graph part of her trip, first click the correct line type in the box. Then click in the graph to add the starting point and the ending point for that part of her trip. Repeat these steps until a graph of Carla's entire trip has been created.

Graph each part of Carla's trip. To graph part of her trip, first click the correct line type in the box. Then click in the graph to add the starting point and the ending point for that part of her trip.
Repeat these steps until a graph of Carla's entire trip has been created.





Sample Solution:



Instructional Strategies: See **Grade 8.F.4**

Resources/Tools:

[8.F Tides]
[8.F Distance]
[8.F Bike Race]
[8.F Riding by the Library]

Common Misconceptions: See **Grade 8.F.4**

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Geometry (G)

Cluster: *Understand congruence and similarity using physical models, transparencies, or geometry software.*

Standard: **Grade 8.G.1**

Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP 4 Model with mathematics.
- ✓ MP 5 Use appropriate tools strategically.
- ✓ MP 6 Attend to precision.
- ✓ MP 7 Look for and make use of structure.
- ✓ MP 8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Grade 8 Critical Area of Focus #3: **Analyzing two- and three- dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.**
- This cluster builds from Grade 7 Geometry, Ratios and Proportional Relationships, and prepares students for more formal work in high school geometry.

Explanations and Examples:

In a translation, every point of the pre-image is moved the same distance and in the same direction to form the image. A **reflection** is the "flipping" of an object over a line, known as the "line of reflection". A **rotation** is a transformation that is performed by "spinning" the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise.

Students use compasses, protractors and ruler or technology to explore figures created from translations, reflections and rotations. Characteristics of figures, such as lengths of line segments, angle measures and parallel lines are explored before the transformation (pre-image) and after the transformation (image). Students understand that these transformations produce images of exactly the same size and shape as the pre-image and are known as rigid transformations.

Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated.

Students are not expected to work formally with properties of dilations until high school.

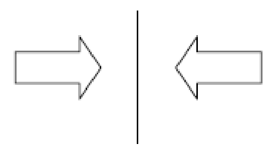
Instructional Strategies:

A major focus in Grade 8 is to use knowledge of angles and distance to analyze two- and three- dimensional figures and space in order to solve problems. This cluster interweaves the relationships of symmetry, transformations, and angle relationships to form understandings of similarity and congruence. Inductive and deductive reasoning are utilized as students forge into the world of proofs. Informal arguments are justifications based on known facts and logical reasoning.

Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). Students are expected to use logical thinking, expressed in words using correct terminology. They are NOT expected to use theorems, axioms, postulates or a formal format of proof as in two-column proofs.

Transformational geometry is about the effects of rigid motions, rotations, reflections and translations on figures. Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. For example, when reflecting over a line, each vertex is the same distance from the line as its corresponding vertex. This is easier to visualize when not using regular figures. Time should be allowed for students to cut out and trace the figures for each step in a series of transformations. Discussion should include the description of the relationship between the original figure and its image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, including the attributes of transformations (line of symmetry, distance to be moved, center of rotation, angle of rotation and the amount of dilation). The case of distance – preserving transformation leads to the idea of congruence.

It is these distance-preserving transformations that lead to the idea of congruence.



Work in the coordinate plane should involve the movement of various polygons by addition, subtraction and multiplied changes of the coordinates. For example, add 3 to *x*, subtract 4 from *y*, combinations of changes to *x* and *y*, multiply coordinates by 2 then by $\frac{1}{2}$. Students should observe and discuss such questions as "What happens to the polygon?" and "What does making the change to all vertices do?" Understandings should include generalizations about the changes that maintain size or maintain shape, as well as the changes that create distortions of the polygon (dilations). Example dilations should be analyzed by students to discover the movement from the origin and the subsequent change of edge lengths of the figures. Students should be asked to describe the transformations required to go from an original figure to a transformed figure (image). Provide opportunities for students to discuss the procedure used, whether different procedures can obtain the same results, and if there is a more efficient procedure to obtain the same results. Students need to learn to describe transformations with both words and numbers.

Through understanding symmetry and congruence, conclusions can be made about the relationships of line segments and angles with figures. Students should relate rigid motions to the concept of symmetry and to use them to prove congruence or similarity of two figures. Problem situations should require students to use this knowledge to solve for missing measures or to prove relationships. It is an expectation to be able to describe rigid motions with coordinates.

Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures, for example, the corresponding angles of similar figures are equal. Additionally use drawings of parallel lines cut by a transversal to investigate the relationship among the angles.

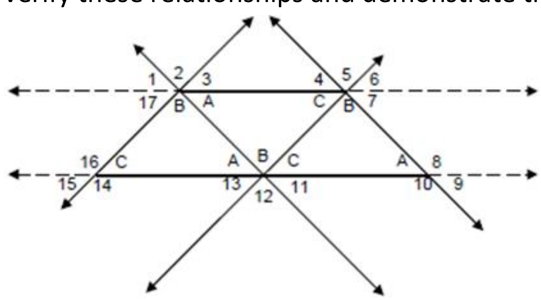
For example, what information can be obtained by cutting between the two intersections and sliding one onto the other?

In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, vertical). Now, the focus is on learning the about the sum of the angles of a triangle and using it to, find the measures of angles formed by transversals (especially with parallel lines), or to find the measures of exterior angles of triangles and to informally prove congruence.

By using three copies of the same triangle labeled and placed so that the three different angles form a straight line, students can:

- explore the relationships of the angles,
- learn the types of angles (interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, same side exterior)
- explore the parallel lines, triangles and parallelograms formed

Further examples can be explored to verify these relationships and demonstrate their relevance in real life.



Investigations should also lead to the Angle-Angle criterion for similar triangles. For instance, pairs of students create two different triangles with one given angle measurement, then repeat with two given angle measurements and finally with three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.

Students should solve mathematical and real-life problems involving understandings from this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist in the more formal learning of geometry in high school.

Resources/Tools:
8.G Reflecting a rectangle over a diagonal
8.G, G-GPE, G-SRT, G-CO Is this a rectangle?
8.G Partitioning a hexagon
8.G Same Size, Same Shape?
8.G, G-CO Origami Silver Rectangle

Common Misconceptions:
Students confuse the rules for transforming two-dimensional figures because they rely too heavily on rules as opposed to understanding what happens to figures as they translate, rotate, reflect, and dilate. It is important to have students describe the effects of each of the transformations on two-dimensional figures through the coordinates but also the visual transformations that result.

By definition, congruent figures are also similar. It is incorrect to say that similar figures are the same shape, just a different size. This thinking leads students to misconceptions such as that all triangles are similar. It is important to add to that definition, the property of proportionality among similar figures.

Student errors with the Pythagorean theorem may involve mistaking one of the legs as the hypotenuse, multiplying the legs and the hypotenuse by 2 as opposed to squaring them, or using the theorem to find missing sides for a triangle that is not right.

Students often confuse situations that require adding with multiplicative situations in regard to scale factor. Providing experiences with geometric figures and coordinate grids may help students visualize the different.

Students have difficulty differentiating between congruency and similarity. Assume any combination of three angles will form a congruence condition.

Not recognize congruent figures because of different orientations.

Confuse terms such as clockwise and counter-clockwise. Think the line of reflection must be vertical or horizontal (e.g. across the y- axis or x-axis. Not realize that rotations are not always the origin, but can be about any point.

Domain: Geometry (G)

Cluster: *Understand congruence and similarity using physical models, transparencies, or geometry software.*

Standard: **Grade 8.G.2**

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure
- ✓ MP.8 Look for and express regularity in repeated reasoning

Connections: See **Grade 8.G.1**

Explanations and Examples:

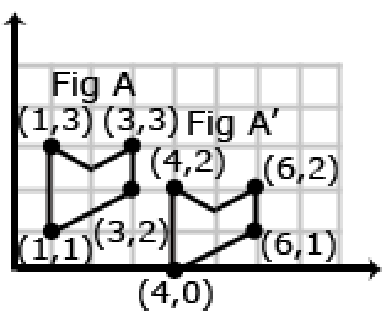
This standard is the students' introduction to congruency. Congruent figures have the same shape and size. Translations, reflections and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent).

Students examine two figures to determine congruency by identifying the rigid transformation(s) that produced the figures. Students recognize the symbol for congruency (≅) and write statements of congruency.

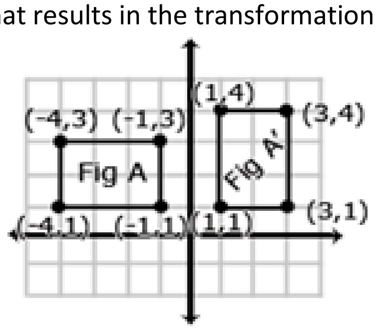
Examples:

Is Figure A congruent to Figure A'?

Explain how you know.

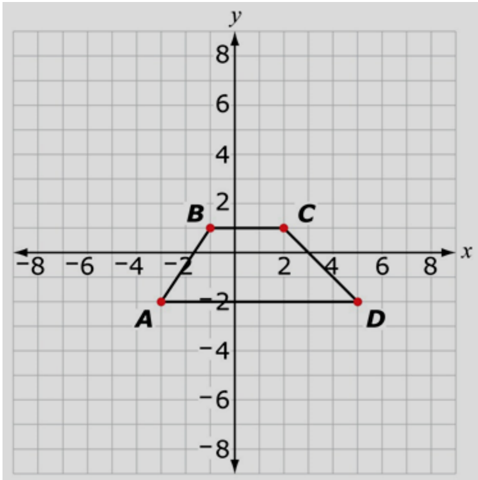


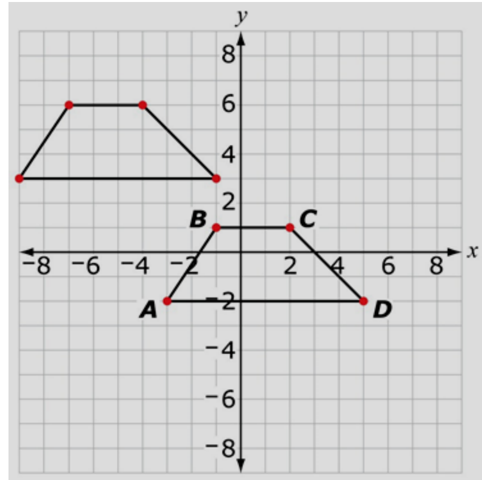
Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



Trapezoid *ABCD* is shown on this coordinate grid. Translate trapezoid *ABCD* 6 units to the left and 5 units up and graph the image of *ABCD* on the grid.

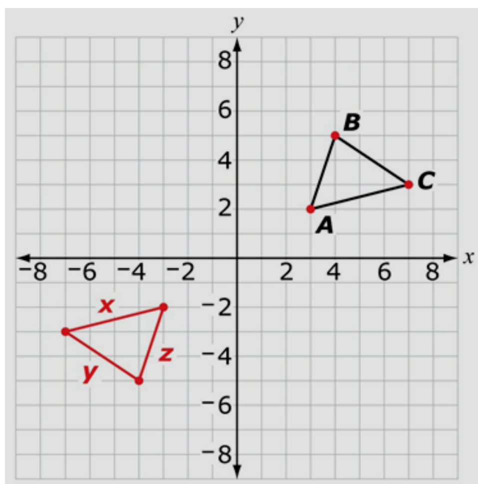
Solution:





Triangle *ABC* on this coordinate grid was created by joining points *A* (3, 2), *B* (4, 5), and *C* (7, 3) with line segments.

Triangle *ABC* was reflected over the *x*-axis and then reflected over the *y*-axis to form the red triangle, where *x*, *y*, and *z* represent the lengths of the sides of the red triangle.



Mark the appropriate boxes in the table to show which sides of the triangles have equal lengths.

	x	***y***	***z***
AB			
AC			
BC			

Solution:

AB = *z*

AC = *x*

BC = *y*

Instructional Strategies: See **Grade 8.G.1**

Major Supporting Additional **Depth Opportunities(DO)**

Resources/Tools:

8.G Congruent Segments

8.G Congruent Rectangles

8.G Congruent Triangles

8.G Triangle congruence with coordinates

8.G Cutting a rectangle into two congruent triangles

8.G Circle Sandwich

Common Misconceptions: See **Grade 8.G.1**

Domain: Geometry (G)

Cluster: *Understand congruence and similarity using physical models, transparencies, or geometry software.*

Standard: **Grade 8.G.3**

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.G.1**

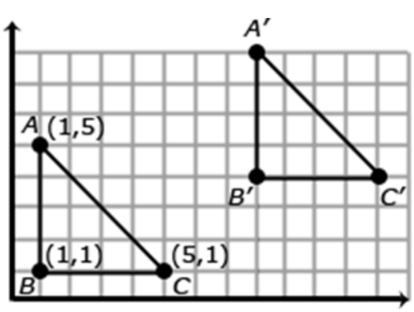
Explanations and Examples:

Students identify resulting coordinates from translations, reflections, and rotations (90°, 180°and 270° both clockwise and counterclockwise), recognizing the relationship between the coordinates and the transformation. For example, a translation of 5 left and 2 up would subtract 5 from the *x*-coordinate and add 2 to the y-coordinate. $D(-4,-3) \rightarrow D'(-9,-1)$. A reflection across the *x*-axis would change $(6,-8) \rightarrow B'(6,8)$.

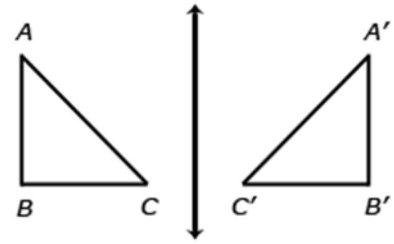
Additionally, students recognize the relationship between the coordinates of the pre-image, the image and the scale factor following a dilation from the origin. Dilations are non-rigid transformations that enlarge (scale factors greater than one) or reduce (scale factors less than one) the size of a figure using a scale factor.

A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is *similar* to its pre-image.

Translation: A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. ΔABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from *x* = 1 to *x* = 8) and 3 units up (from *y* = 5 to *y* = 8). Points B and C also move in the same direction (7 units to the right and 3 units up).

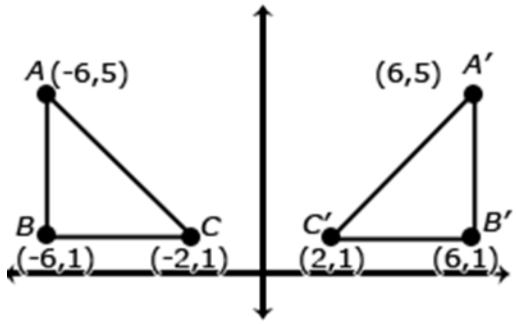


Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is *congruent* to its pre-image.



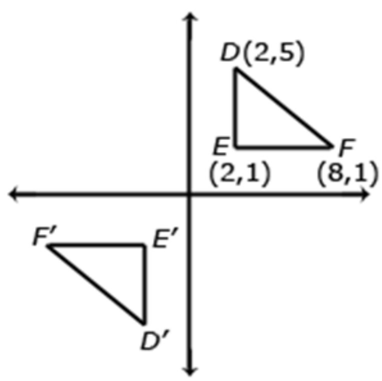
$\triangle ABC \cong \triangle A'B'C'$

When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate.



Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360˚. Rotated figures are congruent to their pre-image figures.

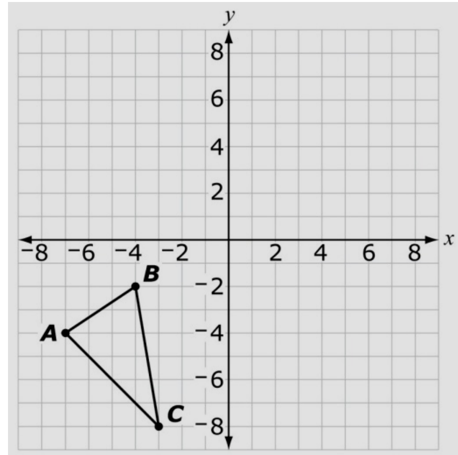
Consider when ΔDEF is rotated 180˚ clockwise about the origin. The coordinates of ΔDEF are D(2,5), E(2,1), and F(8,1). When rotated 180˚, $\Delta D'E'F'$ has new coordinates D'(-2,-5), E'(-2,-1) and F'(-8,-1). Each coordinate is the opposite of its pre-image.



Major Supporting Additional **Depth Opportunities(DO)**

Examples*:*

Triangle ABC is shown on this coordinate grid.



Part A

ΔABC is rotated 180 degrees clockwise about the origin to form ΔDEF.

What are the coordinates of the vertices of ΔDEF?

D(☐ ☐), E(☐ ☐), F(☐ ☐)

Part B

What conjecture can be made about the relationship between the coordinates of the vertices of an original shape and the coordinates of the vertices of the image of the shape when it is rotated 180 degrees clockwise about the origin?

You must demonstrate that the conjecture is always true or that there is at least one example in which the conjecture is not true.

Sample Response:

Part A

D (7, 4), E (4, 2), F (3, 8)

Part B

The conjecture is that the coordinates of the vertices of the image will have the opposite sign of the coordinates of the vertices of the original shape. When a point is rotated 180 degrees clockwise about the origin, if a line is drawn through the original point and the origin, the image of the point will also be on the line, and it will be the same distance from the origin that the original point was, but on the opposite side of the origin. When two points on the same line are the same distance from the origin and on opposite sides of the origin, the coordinates of the points have opposite signs, because the slope from each coordinate to the origin is the same, but to move from the origin to each point to get its coordinates, you must move in opposite directions. So if you move right from the origin to get to one point, you will move left to get to the other, and if you move up from the origin to get to one point, you will move down to get to the other. So the coordinates of the vertices of the image will have the opposite sign of the coordinates of the vertices of the original shape.

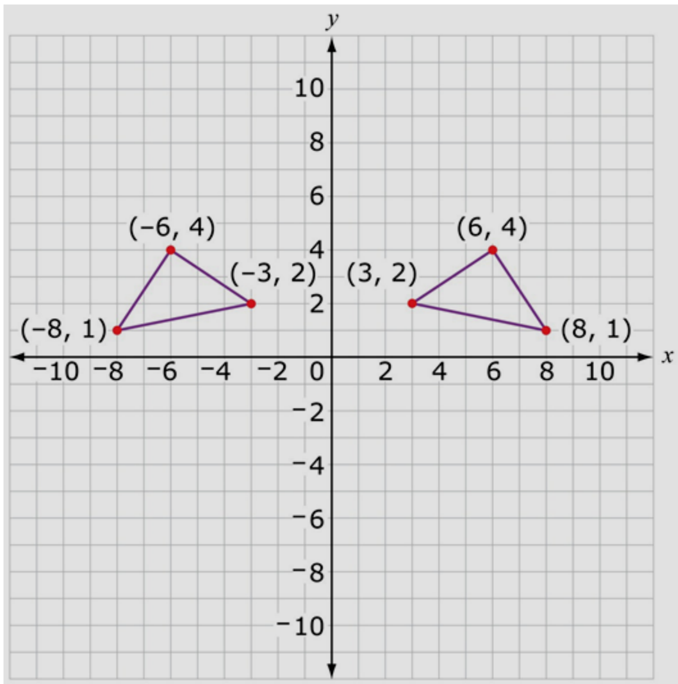
A student made this conjecture about reflections on an *xy*-coordinate plane.

When a polygon is reflected over the *y*-axis, the *x*-coordinates of the corresponding vertices of the polygon and its image are opposite, but the *y*-coordinates are the same.

Develop a chain of reasoning to justify or refute the conjecture. You must demonstrate that the conjecture is always true or that there is at least one example in which the conjecture is not true. You may include one or more graphs in your response.

Major Supporting Additional **Depth Opportunities(DO)**

Sample Response:
When a polygon is reflected over the *y*-axis, each vertex of the reflected polygon will end up on the opposite side of the *y*-axis but the same distance from the *y*-axis. So, the *x*- coordinates of the vertices will change from positive to negative or negative to positive, but the absolute value of the number will stay the same, so the *x*-coordinates of the corresponding vertices of the polygon and its image are opposites. Since the polygon is being reflected over the *y*-axis, the image is in a different place horizontally but it does not move up or down, which means the *y*-coordinates of the vertices of the image will be the same as the *y*-coordinates of the corresponding vertices of the original polygon.
As an example, look at the graph below, and notice that the *x*-coordinates of the corresponding vertices of the polygon and its image are opposites but the *y*-coordinates are the same.
This means the conjecture is correct.



Instructional Strategies: See **Grade 8.G.1**

Resources/Tools:
[8.G Reflecting reflections]
[8.G Triangle congruence with coordinates]
[8.G Point Reflection]
[8.G.A.3 Effects of Dilations on Length, Area, and Angles]

Common Misconceptions: See **Grade 8.G.1**

Domain: Geometry (G)

Cluster: *Understand congruence and similarity using physical models, transparencies, or geometry software.*

Standard: **Grade 8.G.4**

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them
- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

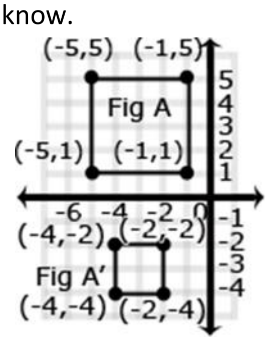
Connections: See **Grade 8.G.1**

Explanations and Examples:

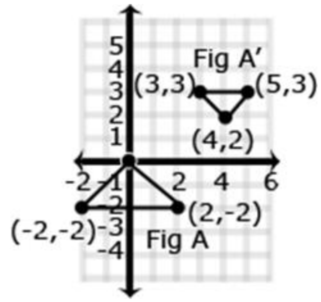
This is the students' introduction to similarity and similar figures. Students understand similar figures have angles with the same measure and sides that are proportional. Similar figures are produced from dilations. Students describe the sequence that would produce similar figures, including the scale factors. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size.

Examples:

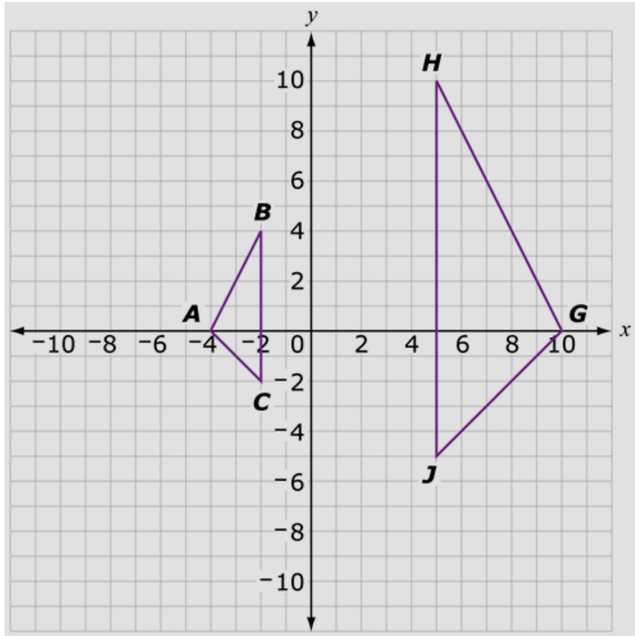
Is Figure A similar to Figure A'? Explain how you know.



Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



A transformation is applied to ΔABC to form ΔDEF (not shown). Then, a transformation is applied to ΔDEF to form ΔGHJ.



Part A
Graph ΔDEF on the xy-coordinate plane.

Part B
Describe the transformation applied to ΔDEF to form ΔABC.

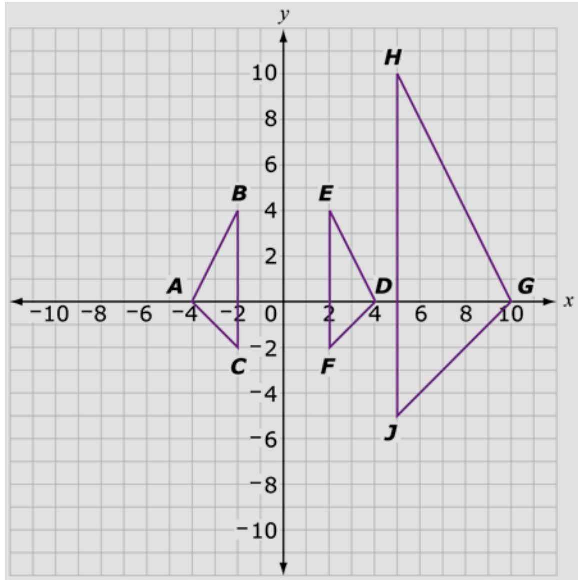
Part C
Describe the transformation applied to ΔDEF to form ΔGHJ.

Part D
Select one statement that applies to the relationship between Δ*GHJ* and Δ*ABC*.

- ΔGHJ is congruent to ΔABC.
- ΔGHJ is similar to ΔABC.
- ΔGHJ is neither congruent nor similar to ΔABC.

Explain your reasoning.

Sample Response:
Part A



Part B
A reflection over the *y*-axis

Part C
A dilation with a scale factor of 2.5 about the origin.

Part D
ΔGHJ is similar to ΔABC.

A dilation followed by a congruence, or a congruence followed by a dilation, is a similarity.
So, ΔGHJ is similar to ΔABC.

Instructional Strategies: See **Grade 8.G.1**

Resources/Tools:
8.G.A.4 Are They Similar?
8.G Creating Similar Triangles

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

Common Misconceptions: See **Grade 8.G.1**

Domain: Geometry (G)

Cluster: *Understand congruence and similarity using physical models, transparencies, or geometry software.*

Standard: Grade 8.G.5

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them
- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.G.1**

Explanations and Examples:

Students use exploration and deductive reasoning to determine relationships that exist between a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle- angle criterion for similarity of triangle.

Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle. (The measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360°). Using these relationships, students use deductive reasoning to find the measure of missing angles.

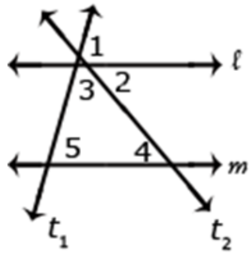
Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles.

Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar.

Students can informally prove relationships with transversals.

Examples:

Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180^{\circ}$ if l and m are parallel lines and t_1 & t_2 are transversals.

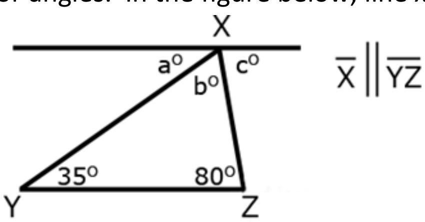


Sample Response:

$\angle 1 + \angle 2 + \angle 3 = 180^{\circ}$. Angle 1 and Angle 5 are congruent because they are corresponding angles ($\angle 5 \cong \angle 1$). $\angle 1$ can be substituted for $\angle 5$. $\angle 4 \cong \angle 2$: because alternate interior angles are congruent. $\angle 4$ can be substituted for $\angle 2$

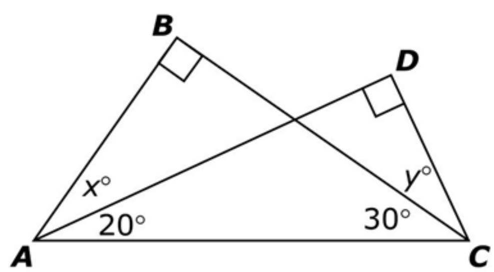
Therefore $m\angle 3 + m\angle 4 + m\angle 5 = 180^{\circ}$

Students can informally conclude that the sum of a triangle is 180º (the angle-sum theorem) by applying their understanding of lines and alternate interior angles. In the figure below, line x is parallel to line *yz*:



Angle *a* is 35° because it alternates with the angle inside the triangle that measures 35°. Angle *c* is 80° because it alternates with the angle inside the triangle that measures 80°. Because lines have a measure of 180°, and angles *a* + *b* + *c* form a straight line, then angle *b* must be 65° (180 – 35 + 80 = 65). Therefore, the sum of the angles of the triangle are $35^{\circ} + 65^{\circ} + 80^{\circ}$

Right triangle *ABC* and right triangle *ACD* overlap as shown below. Angle *DAC* measures 20º and angle *BCA* measures 30º.



not drawn to scale

What are the values of *x* and *y*?

Solution:

x = 40 and *y* = 40

Students need to use the fact that the sum of the angles of a triangle is 180 degrees to find the correct values of *x* and *y*. Students may incorrectly assume that *x* + 20 must equal *y* + 30.

Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies: See **Grade 8.G.5**

Resources/Tools:

8.G Find the Angle

8.G Find the Missing Angle

8.G Tile Patterns II: hexagons

8.G Tile Patterns I: octagons and squares

8.G A Triangle's Interior Angles

8.G Congruence of Alternate Interior Angles via Rotations

8.G.A.5 Street Intersections

8.G Rigid motions and congruent angles

Common Misconceptions: See **Grade 8.G.1**

Domain: Geometry (G)

Cluster: *Understand and apply the Pythagorean Theorem.*

Standard: Grade 8.G.6

Explain a proof of the Pythagorean Theorem and its converse.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP 1 Solve problems and persevere in solving them.
- ✓ MP 3 Construct viable arguments and critique the reasoning of others.
- ✓ MP 4 Model with mathematics.
- ✓ MP 6 Attend to precision
- ✓ MP 7 Look for and make use of structure.

Connections:

This cluster is connected to:

- Grade 8 Critical Area of Focus #3: **analyzing two- and three- dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.**

Explanations and Examples:

Students explain the Pythagorean Theorem as it relates to the area of squares coming off of all sides of a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle.

Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle.

Instructional Strategies:

Previous understanding of triangles, such as the sum of two side measures is greater than the third side measure, angles sum, and area of squares, is furthered by the introduction of unique qualities of right triangles. Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Data should be recorded in a chart such as the one below, allowing for students to conjecture about the relationship among the areas within each triangle.

Triangle	Measure of Leg 1	Measure of Leg 2	Area of Square on Leg 1	Area of Square on Leg 2	Area of square on Hypotenuse
1					

Students should then test out their conjectures, then explain and discuss their findings. Finally, the Pythagorean Theorem should be introduced and explained as the **pattern** they have explored. Time should be spent analyzing several proofs of the Pythagorean Theorem to develop a beginning sense of the process of deductive reasoning, the significance of a theorem, and the purpose of a proof. Students should be able to justify a simple proof of the Pythagorean Theorem or its converse.

Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need situations that explore using the Pythagorean Theorem to test whether or not side lengths represent right triangles. (Recording could include Side length *a*, Side length *b*, Sum of $a^2 + b^2$, c^2, $a^2 + b^2 = c^2$, Right triangle? Through these opportunities, students should realize that there are Pythagorean (triangular) triples such as (3, 4, 5), (5, 12, 13), (7, 24, 25), (9, 40, 41) that always create right triangles, and that their multiples also form right triangles. Students should see how similar triangles can be used to find additional triples. Students should be able to explain why a triangle is or is not a right triangle using the Pythagorean Theorem.

The Pythagorean Theorem should be applied to finding the lengths of segments on a coordinate grid, especially those segments that do not follow the vertical or horizontal lines, as a means of discussing the determination of distances between points. Contextual situations, created by both the students and the teacher, that apply the Pythagorean Theorem and its converse should be provided.

For example, apply the concept of similarity to determine the height of a tree using the ratio between the student's height and the length of the student's shadow. From that, determine the distance from the tip of the tree to the end of its shadow and verify by comparing to the computed distance from the top of the student's head to the end of the student's shadow, using the ratio calculated previously.

Challenge students to identify additional ways that the Pythagorean Theorem is or can be used in real world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the diagonal of a prism.

Tools/Resources:

8.G Applying the Pythagorean Theorem in a mathematical context
8.G Bird and Dog Race
8.G, G-GPE, G-SRT, G-CO Is this a rectangle?
8.G A rectangle in the coordinate plane
8.G.B Sizing up Squares
8.G Converse of the Pythagorean Theorem
Maryland Geometry Lessons
Video Tutorial with Graph Paper (Pythagorean Theorem with 3-4-5 Triangle

Domain: Geometry (G)

Cluster: *Understand and apply the Pythagorean Theorem.*

Standard: Grade 8.G.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them
- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure

Connections: See **Grade 8.G.6**; **Grade 8.NS.2**

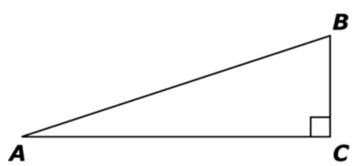
Explanations and Examples:

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets.

Examples:

In right triangle *ABC*, side *AC* is longer than side *BC*. The boxed numbers represent the possible side lengths of triangle *ABC*.



not drawn to scale

7	**8**
15	**17**
18	**20**
24	**25**

Identify three boxed numbers that could be the side lengths of triangle *ABC*.

1a. *BC* =

1b. *AC* =

1c. *AB* =

Solutions: BC = 7, AC = 24, AB = 25 **or** BC = 15, AC = 20, AB = 25 **or** BC = 8, AC = 15, AB = 17

Part A
Triangle *STV* has sides with lengths of 7, 11, and 14 units. Determine whether this triangle is a right triangle. Show all work necessary to justify your answer.

Part B
A right triangle has a hypotenuse with a length of 15. The lengths of the legs are whole numbers. What are the lengths of the legs?

Sample Response:
Part A
72 + 112 does not equal 142 because 49 + 121 = 170, not 196.
Therefore, it is not a right triangle because the side lengths do not satisfy the Pythagorean Theorem.

Part B
9, 12

Students in a class are using their knowledge of the Pythagorean Theorem to make conjectures about triangles. A student makes the conjecture shown below.

A triangle has side lengths x, y, z. If $x < y < z$ and $x^2 + y^2 < z^2$, the triangle is an obtuse triangle.

Use the Pythagorean Theorem to develop a chain of reasoning to justify or refute the conjecture. You must demonstrate that the conjecture is always true or that there is at least one example in which the conjecture is not true.

Sample Response:
Picture the triangle with the side of length x on the bottom, the side of length y on the left, and the side of length z on the top. If $x^2 + y^2 = z^2$ the triangle is a right triangle. Since $x^2 + y^2 < z^2$ if the sides of length x and y were left so they made a right angle and the side of length z started at the other end of the side of length x, it would extend past the other end of the side of length y. So the end of the side of length y has to swing out to the left so the ends of all the segments can connect to form a triangle. When the side of length y swings out to the left, the measure of the angle between that side and the side of length x increases, so the triangle is an obtuse triangle. The conjecture is true.

Instructional Strategies: See **Grade 8.G.6**

Resources/Tools:
8.G Glasses
8.G Spiderbox
8.G.7 Running on the Football Field
8.G Two Triangles' Area
8.G Area of a Trapezoid
8.G, G-SRT Points from Directions
8.G Areas of Geometric Shapes with the Same Perimeter 8.G Circle Sandwich

Pythagorean Lessons: CCGPS Frameworks Geometric Applications of Exponents

Domain: Geometry (G)

Cluster: *Understand and apply the Pythagorean Theorem.*

Standard: Grade 8.G.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them
- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure

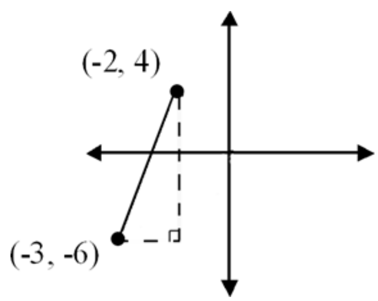
Connections: See **Grade 8.G.6**; **Grade 8.NS.2**

One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students build on work from 6th grade (finding vertical and horizontal distances on the coordinate plane) to determine the lengths of the legs of the right triangle drawn connecting the points. Students understand that the line segment between the two points is the length of the hypotenuse. The use of the distance formula is not an expectation.

Explanations and Examples:

Examples:

Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.



What is the distance between (0, 0) and (8, 15) on the *xy*- coordinate plane?

Solution: 17 units

Common errors:

Students may subtract 8 from 15 to find the distance. Students may think that the distance is equal to the *x* value of the point that is not at the origin. Students may add 8 and 15 to find the distance.

Instructional Strategies: See **Grade 8.G.6**

Resources/Tools:

[8.G Finding isosceles triangles]

[8.G Finding the distance between points]

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Geometry (G)

Cluster: *Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.*

Standard: Grade 8.G.9

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Solve problems and persevere in solving them
- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

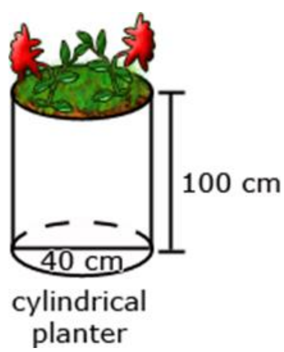
- Grade 8 Critical Area of Focus #3: **Analyzing two- and three- dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.**

Explanations and Examples:

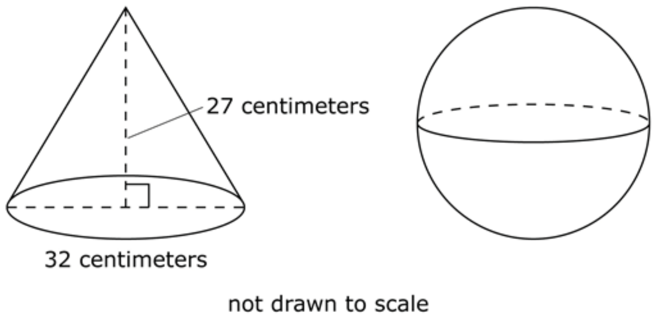
Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, cones and spheres. Students understand the relationship between the volume of a) cylinders and cones and b) cylinders and spheres to the corresponding formulas.

Examples:

James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.



This cone and sphere have equal volumes.



What is the radius of the sphere?

Solution: 12 centimeters

A cylindrical tank has a height of 10 feet and a radius of 4 feet. Jane fills the tank with water at a rate of 8 cubic feet per minute. At this rate, how many minutes will it take Jane to completely fill the tank without overflowing it? Round your answer to the nearest **minute**.

Solution: 63 minutes

Juan needs a right cylindrical storage tank that holds between 110 and 115 cubic feet of water. Using whole numbers only, provide the radius and height for 3 different tanks that hold between 110 and 115 cubic feet of water.

Tank #1	
radius	ft.
height	ft.

Tank #2	
radius	ft.
height	ft.

Tank #3	
radius	ft.
height	ft.

Sample Response:

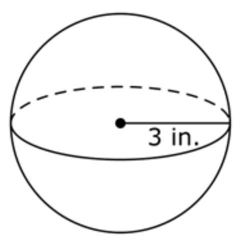
Tank #1	
radius	2 ft.
height	9 ft.

Tank #2	
radius	3 ft.
height	4 ft.

Tank #3	
radius	6 ft.
height	1 ft.

Part A

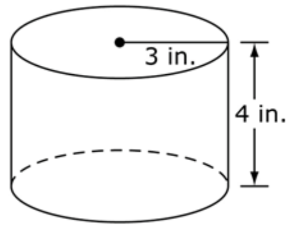
This sphere has a 3-inch radius.



What is the volume, in cubic inches, of the sphere?

Part B

This right cylinder has a radius of 3 inches and a height of 4 inches.



What is the volume, in cubic inches, of the cylinder?

Part C

Lin claims that the volume of any sphere with a radius of *r* inches is always **equal** to the volume of a cylinder with a radius of *r* inches and a height of *h* inches, when $h = \frac{4}{3}r$. Show all work necessary to justify Lin's claim.

Sample Response:

Part A: 36π cu in. (or any number between 113 and 113.1)

Part B: 36π cu in. (or any number between 113 and 113.1)

Part C: I can create the following equation if the volume of the sphere and cylinder are equal,

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

I can divide both sides of the equation by (πr^2) as shown below.

$$\left(\frac{4}{3}\pi r^3\right) \div (\pi r^2) = (\pi r^2 h) \div \pi r^2$$

This justifies Lin's claim. $\frac{4}{3}r = h$

Instructional Strategies:

Begin by recalling the formula, and its meaning, for the volume of a right rectangular prism: $V = l \times w \times h$. Then ask students to consider how this might be used to make a conjecture about the volume formula for a cylinder

Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a base times the height, and so because the area of the base of a cylinder is πr^2 the volume of a cylinder is $V_c = \pi r^2 h$.

To motivate the formula for the volume of a cone, use cylinders and cones with the same base and height. Fill the cone with rice or water and pour into the cylinder. Students will discover/experience that 3 cones full are needed to fill the cylinder. This non-mathematical derivation of the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$, will help most students remember the formula.

In a drawing of a cone inside a cylinder, students might see that that the triangular cross-section of a cone is $\frac{1}{2}$ the rectangular cross-section of the cylinder. Ask them to reason why the volume (three dimensions) turns out to be *less* than $\frac{1}{2}$ the volume of the cylinder. It turns out to be $\frac{1}{3}$.

For the volume of a sphere, it may help to have students visualize a hemisphere "inside" a cylinder with the same height and base.‖ The radius of the circular base of the cylinder is also the radius of the sphere and the hemisphere. The area of the base of the cylinder and the area of the section created by the division of the sphere into a hemisphere is πr^2. The height of the cylinder is also *r* so the volume of the cylinder is πr^3. Students can see that the volume of the hemisphere is less than the volume of the cylinder and more than half the volume of the cylinder. Illustrating this with concrete materials and rice or water will help students see the relative difference in the volumes. At this point, students can reasonably accept that the volume of the hemisphere of radius *r* is $\frac{2}{3}\pi r^3$ and therefore volume of a sphere with radius *r* is twice that or $\frac{4}{3}\pi r^3$. There are several websites with explanations for students who wish to pursue the reasons in more detail. (Note that in the pictures above, the hemisphere and the cone together fill the cylinder.)

Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions.

Resources/Tools:

8.G Comparing Snow Cones
8.G Glasses
8.G Flower Vases
8.G Shipping Rolled Oats

See also Illuminations: Cubes.

Common Misconceptions:

A common misconception among middle grade students is that volume is a number that results from substituting other numbers into a formula. For these students there is no recognition that volume is a measure related to the amount of space occupied. If a teacher discovers that students do not have an understanding of volume as a measure of space, it is important to provide opportunities for hands on experiences where students fill three dimensional objects. Begin with right rectangular prisms and fill them with cubes will help students understand why the units for volume are cubed.

Domain: Statistics and Probability (SP)

Cluster: *Investigate patterns of association in bivariate data.*

Standard: Grade 8.SP.1

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This Cluster is connected to:

- Grade 8 Critical Area of Focus #1: **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.**

Explanations and Examples:

Bivariate data refers to two variable data, one to be graphed on the x-axis and the other on the *y*-axis. Students represent measurement (numerical) data on a scatter plot, recognizing patterns of association. These patterns may be linear (positive, negative or no association) or non-linear.

Students build on their previous knowledge of scatter plots examine relationships between variables.
They analyze scatterplots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets.

Examples:

Data for 10 students' Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

Student	1	2	3	4	5	6	7	8	9	10
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Number of staff	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

Instructional Strategies:

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Scatter plots are the most common form of displaying bivariate data in Grade 8. Provide scatter plots and have students practice informally finding the *line of best fit*. Students should create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. Have students use a graphing calculator to determine a linear regression and discuss how this relates to the graph.

Students should informally draw a line of best fit for a scatter plot and informally measure the strength of fit. Discussion should include “What does it mean to be above the line, below the line?”

The study of the line of best fit ties directly to the algebraic study of slope and intercept.

Students should interpret the slope and intercept of the line of best fit in the context of the data.

Then students can make predictions based on the line of best fit.

Resources/Tools:

For detailed information see Learning Progression for Statistics & Probability:
http://www.learner.org/series/modules/express/videos/video_clips.html?type=1&subject=math&practice=structure

8.SP Birds' Eggs
8-SP.1 Texting and Grades I
8.SP.1 Hand span and height
8.SP Animal Brains

Common Misconceptions:

Students may believe Bivariate data is only displayed in scatter plots. Grade 8.SP.4 in this cluster provides the opportunity to display bivariate, categorical data in a table.

In general, students think there is only one correct answer in mathematics. Students may mistakenly think their lines of best fit for the same set of data will be exactly the same. Because students are informally drawing lines of best fit, the lines will vary slightly. To obtain the exact line of best fit, students would use technology to find the line of regression.

Domain: Statistics and Probability (SP)

Cluster: *Investigate patterns of association in bivariate data.*

Standard: Grade 8.SP.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.SP.1**; **Grade 8.EE.5**; **Grade 8.F.3**

Explanations and Examples:

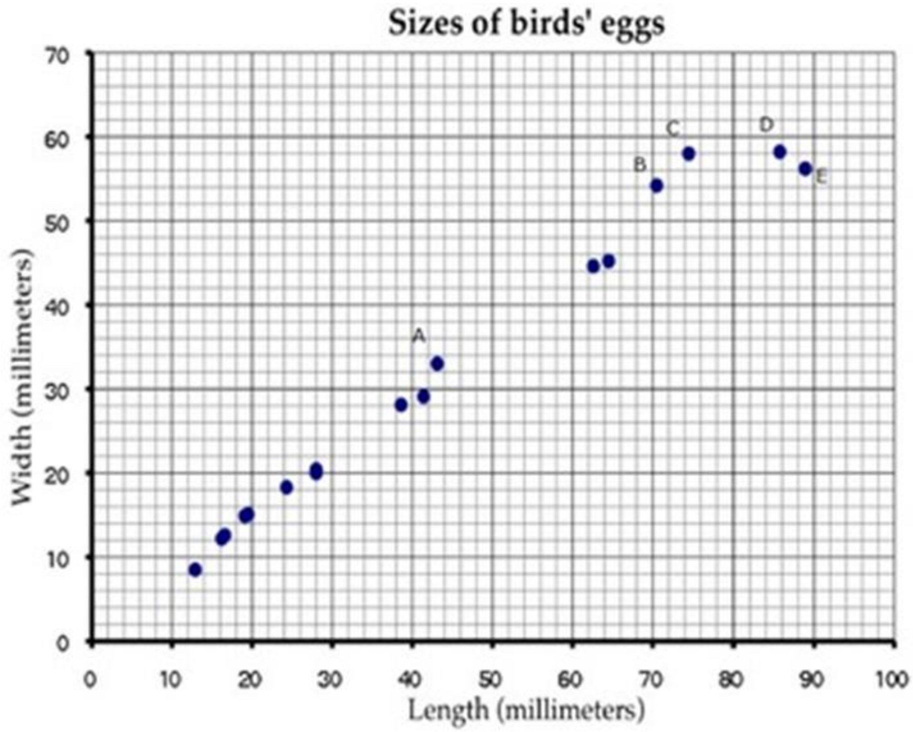
Students understand that a straight line can represent a scatter plot with linear association. The most appropriate linear model is the line that comes closest to most data points. The use of linear regression is not expected.

Examples:

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles Traveled	0	75	120	160	250	300
Gallons Used	0	2.3	4.5	5.7	9.7	10.7

This scatter diagram shows the lengths and widths of the eggs of some American birds.



1. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.

2. What does the graph show about the relationship between the lengths of birds' eggs and their widths?

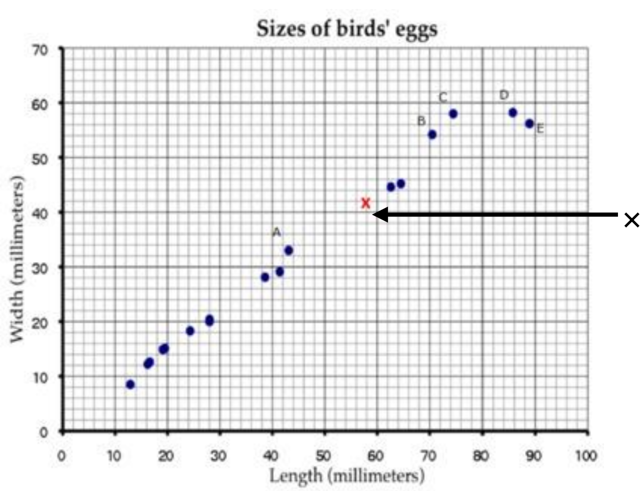
3. Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?

4. Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.

5. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

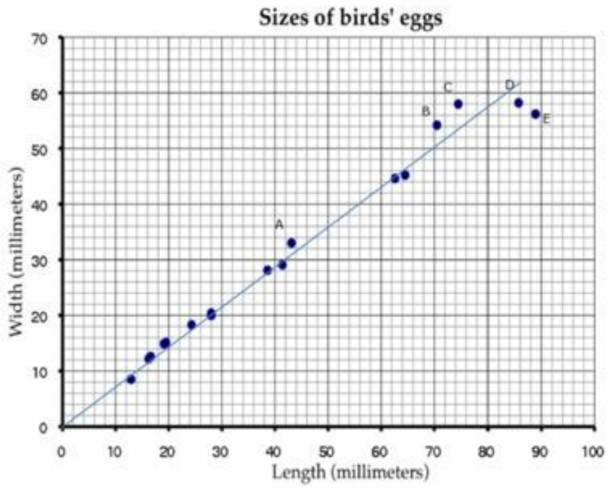
Sample Responses:

1.



Major Supporting Additional **Depth Opportunities(DO)**

2. There seems to be a positive linear relationship between the length and width of the eggs.
3. The line below appears to fit the data fairly well: (Connects to 8.SP.2)



Since it passes through (0,0) and (50,36), it's slope is $\frac{36}{50} = 0.72$, so the equation of the line is *y*=0.72*x*

If x = 35, then our line would predict that $y = 0.72 \cdot 35 = 25.2$. So we would expect the width of these eggs to be, on average, about 25 mm. Answers using different lines can vary up to 1 mm in either direction.

4. Without reading off precise numerical values from the plot, we can see that eggs C and D have very nearly the same width, but egg D is about 12 millimeters longer than egg C.

5. First we note that egg E certainly has a higher length-to-width ratio than C or D, since it is both longer and narrowed. Similarly, E has a higher ratio than B because it is significantly longer, and only a tad wider. It is harder to visually identify the difference between A and E, we compute their respective length-to-width ratios numerically, which turn out to be approximately 1.3 for A and 1.6 for E. So E has the greatest ratio of length to width.

Resources/Tools:
8.SP Birds' Eggs
8.SP Animal Brains
8.SP Laptop Battery Charge

Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics

Common Misconceptions: See **Grade 8.SP.1**

Domain: Statistics and Probability (SP)

Cluster: *Investigate patterns of association in bivariate data.*

Standard: Grade 8.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

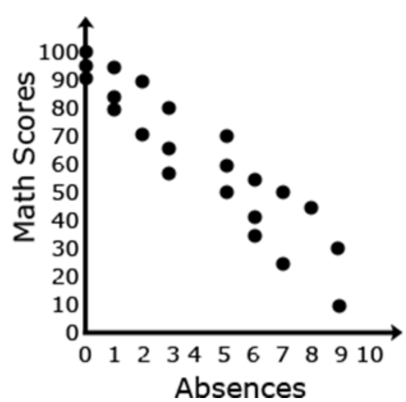
Connections: See **Grade 8.SP.1**; **Grade 8.EE.5**; **Grade 8.F.3**; **Grade 8.F.4**; **Grade 8.EE.5**

Explanations and Examples:

Given data from students' math scores and absences, make a scatterplot.

Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80

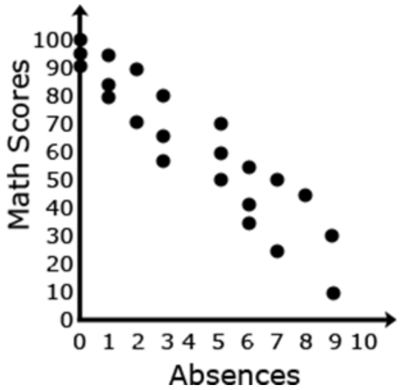
Solution:

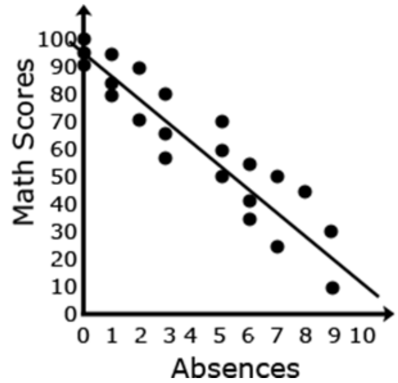


Major Supporting Additional **Depth Opportunities(DO)**

Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.

Solution:





From the line of best fit, determine an approximate linear equation that models the given data.

Solution: (about $y = -\frac{25}{3}x + 95$)

Students should recognize that 95 represents the y intercept and $-\frac{25}{3}$ represents the slope of the line.

Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can then compare this value to their line.

Instructional Strategies:

Scatter plots are the most common form of displaying bivariate data in Grade 8. Provide scatter plots and have students practice informally finding the *line of best fit*. Students should create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. Have students use a graphing calculator to determine a linear regression and discuss how this relates to the graph.

Students should informally draw a line of best fit for a scatter plot and informally measure the strength of fit. Discussion should include "What does it mean to be above the line, below the line?"

The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students should interpret the slope and intercept of the line of best fit in the context of the data. Then students can make predictions based on the line of best fit.

Resources/Tools:

8.SP US Airports, Assessment Variation

"Walk the Graph", Georgia Department of Education.

In this task, students will use CBR™ motion detectors to create real-time graphs displaying lines with positive, negative, and zero slopes.

Common Misconceptions: See **Grade 8.SP.1**

Domain: Statistics and Probability (SP)

Cluster: *Investigate patterns of association in bivariate data*.

Standard: Grade 8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP 2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 8.SP.1**

Explanations and Examples: 8.SP.4

Students recognize that categorical data can also be described numerically through the use of a two-way table. A two-way table is a table that shows categorical data classified in two different ways.

The frequency of the occurrences are used to identify possible associations between the variables. For example, a survey was conducted to determine if boys eat breakfast more often than girls.

The following table shows the results:

	Male	**Female**
Eat breakfast on a regular basis	190	110
Do not eat breakfast on a regular basis	130	165

Students can use the information from the table to compare the probabilities of males eating breakfast (190 of the 320 males = 59%) and females eating breakfast (110 of the 375 females = 29%) to answer the question. From this data, it can be determined that males do eat breakfast more regularly than females.

Examples:

The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew? and Do you have assigned chores?

Is there evidence that those who have a curfew also tend to have chores?

		Curfew	
		Yes	No
Chores	Yes	40	10
	No	10	40

Solution:

Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.

Jacob surveyed 25 adults to ask whether they had at least one child under the age of 18 and whether they had at least one pet. This table shows the results of the survey.

Adult	At Least One Child Under the Age of 18		At Least One Pet	
	Yes	No	Yes	No
A	✓		✓	
B		✓		✓
C	✓			✓
D		✓		✓
E		✓	✓	
F		✓		✓
G		✓	✓	
H	✓		✓	
I	✓		✓	
J	✓			✓
K		✓		✓
L	✓		✓	
M		✓	✓	
N	✓		✓	
O		✓		✓
P	✓		✓	
Q		✓		✓
R	✓		✓	
S		✓		✓
T	✓		✓	
U		✓	✓	
V	✓		✓	
W		✓		✓
X		✓	✓	
Y		✓	✓	

Part A

Use the results of the survey to complete this table.

At Least One Pet	**At Least One Child Under the Age of 18**	
	Yes	**No**
Yes		
No		

Part B

Jacob made the conjecture that there is a possible association between whether an adult has at least one child under the age of 18 and whether the adult has at least one pet.

State whether the results of the survey provide evidence that adults who have at least one child under the age of 18 also tend to have at least one pet. Explain your answer.

Sample Response:

Part A:

At Least One Pet	At Least One Child Under the Age of 18	
	Yes	No
Yes	9	6
No	2	8

Part B:

Yes, there is evidence that the adults who have at least one child under the age of 18 also tend to have at least one pet. I found the relative frequencies for whether the adult had at least one pet or not given that the adult had at least one child and then given that the adult did not have any children, and 82% of the adults who had at least one child also had at least one pet. My work is shown below.

At Least One Pet	At Least One Child Under the Age of 18		Totals
	Yes	No	
Yes	$\frac{9}{11} \approx 82\%$	$\frac{6}{14} \approx 43\%$	$\frac{15}{25} = 60\%$
No	$\frac{2}{11} \approx 18\%$	$\frac{8}{14} \approx 57\%$	$\frac{10}{25} = 40\%$

Instructional Strategies: See **Grade 8.SP.1**

Resources/Tools:

8-SP.4 What's Your Favorite Subject?

8.SP.4 Music and Sports

APPENDIX

TABLE 1. Common Addition and Subtraction Situations[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ or $5 = 5 + 0$ $5 = 1 + 4$ or $5 = 4 + 1$ $5 = 2 + 3$ or $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$

[1]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Major Supporting Additional **Depth Opportunities(DO)**

TABLE 2. Common Multiplication and Division Situations[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ And $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*: You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*: You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

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TABLE 3. The Properties of Operations

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every (a) there exists$(-a)$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

TABLE 4. The Properties of Equality[*]

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$ then $b=a$
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$
Addition property of equality	If $a=b$ then $a+c=b+c$
Subtraction property of equality	If $a=b$ then $a-c=b-c$
Multiplication property of equality	If $a=b$ then $a\times c=b\times c$
Division property of equality	If $a=b$ and $c\neq 0$ then $a\div c=b\div c$
Substitution property of equality	If $a=b$ then b may be substituted for *a* in any expression containing *a*.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a<b, a=b, a>b$.
If $a>b$ and $b>c$ then $a>c$
If $a>b$ then $b<a$
If $a>b$ then $-a<-b$
If $a>b$ then $a\pm c>b\pm c$
If $a>b$ and $c>0$ then $a\times c>b\times c$
If $a>b$ and $c<0$ then $a\times c<b\times c$
If $a>b$ and $c>0$ then $a\div c>b\div c$
If $a>b$ and $c<0$ then $a\div c<b\div c$

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Table 6. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

Major Supporting Additional **Depth Opportunities(DO)**

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